REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE

3. REPORT TYPE AND DATES COVERED
FINAL REPORT 15 Jan 93 - 14 Jan 96

4. TITLE AND SUBTITLE
Optical Interconnection for Electronic and Optical Computers

5. FUNDING NUMBERS
61102F
2300/HS

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Alabama A&M University
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AFOSR-TR-96
0264

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NE
110 Duncan Avenue Suite B115
Bolling AFB DC 20332-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
F49620-93-1-0123

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This was a broad multi-year effort aimed at the advancement of modern optical sciences in several areas. This report summarizes solid achievement in multiple areas.

19960614 076

14. SUBJECT TERMS

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	

AFOSR FINAL REPORT

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I. INTRODUCTION

This was a broad multi-year effort aimed at the advancement of modern optical sciences in several areas. This report summarizes solid achievements in multiple areas.

In optical computing/pattern recognition, there were many highlights and two Ph.D's still in the pipeline. Their emphases are on moving away from conventional Fourier processors to syntactic patter recognition (Fourier and neural network based), 2D processing of 1D signals, and nonlinear combining of results from multiple orthogonal filters.

In emergent, self-organizing effects, the primary emphases were on hexagonal and channeling structures.

In lasers/laser materials, we made significant advances in two fields - holographic probing of laser materials and powder lasers.

In very fundamental issues, three papers are of special interest. They concern a fuzzy logic metaphysics of quantum mechanics, a derivation of the Schrodinger and other equations based on fuzzy logic.

We realize that these topics are far too broad to cover coherently in a single report. Accordingly, we have chosen to emphasize new areas of research pioneered here especially in optical pattern recognition. As these subsections are rather long, we treat each independently with its own equation numbers references, etc.

Our accomplishments in the other areas will be identified clearly, and references will be provided for those who care to know more.

II. Optical Pattern Recognition

Many pattern recognition intiatives were taken. The highlight was our work on syntactic pattern recognition which we now summarize.

A novel syntactic approach is introduced to treat particular problems in

pattern recognition. The procedure is implemented by employing optical correlation methods for identifying the various primitives appearing in the input pattern and a fuzzy relational scoring is used to determine their importance. Robust pattern recognition with tolerance to normal variations was demonstrated, indicating an efficient new approach for optical pattern recognition.

We address and illustrate here several approaches to optical syntactic pattern recognition [1]. As syntactic pattern recognition is likely to be unfamiliar to some readers, we begin by placing syntactic and the more familiar statistical pattern recognition in context. Statistical methods are usually applied to solve most pattern recognition problems [2]. On the other hand, relatively little work has been done using the syntactic (or structural) approach to pattern recognition. This is especially true in optics. In statistical pattern recognition, a set of characteristic "features" are extracted from patterns and the assignment of each pattern to a pattern class is made by partitioning the feature space [3]. This approach is ideally suited for simple patterns that can be represented in vector forms, like alphanumeric characters. This kind of recognition is purely quantitative without any structural information. The basic idea of syntactic pattern recognition is to recognize an object not directly, but by its description. Syntactic pattern recognition includes statistical representation in addition to providing a structural description of patterns in terms and location of simpler sub-patterns, or pattern primitives [4]. The pattern primitives are useful features of the image that provide a rich description of the visual scene. This approach can be applied to scene analysis where the patterns under consideration are complex.

Fourier optical pattern recognition can be used to gather data required to make a fuzzy comparison of a pattern with the description of an ideal object. This approach is tolerant to scale change and distortion in the pattern

under consideration [1]. In this letter, we present a new and improved scoring procedure for patterns than that described in [1]. In this method automatic fuzzy relational information is generated and used. The score obtained for each pattern is a number "N" which summarizes which features are present to what certainty, and the spatial relations among them. "N" can be viewed as a measure of the probability that the object is present, or a decision can be made based on the number about the presence of the object. Well-formed patterns will have higher numbers than ill-formed patterns. Our scoring procedure is adaptive to the user's requirements with increased tolerance to in-class variations, such as scale change and distortion.

A computer simulation demonstrates the effectiveness of our scoring method by applying it to some examples drawn from optical character recognition, around which most of the techniques used have been developed. One of the characters used is the letter A. Consider the letter A as being comprised of three primitives or sub-patterns, a_1, a_2, and a_3, as shown in Fig. 1. The scene consisting of the letter A can be described as follows: There is an a_1-like feature above and to the right of an a_2-like feature, as well as above and to the left of an a_3-like feature. And there is an a_2-like feature below and to the left of an a_1-like feature, as well as to the left of an a_3-like feature. The line through a_2 and a_3 is more or less horizontal. Similarly, there is an a_3-like feature below and to the right of an a_1-like feature, as well as to the right of an a_2-like feature. This is a fuzzy description and the scoring procedure provides quantitative evaluations of terms such as a_1-like, and more or less.

Matched filters of the primitives were correlated with different input characters, which were sets of computer generated and handprinted *A*'s, *M*'s, and *U*'s. The letters *AAMU* stand for Alabama A&M University. The handprinted characters were all roughly the same size. Ideally, correlation of a perfect *A* with each of the researched primitives should be characterized by one sharp peak in the image plane [1]. The maximum normalized peak values ought to be ~1. This would not be the case for real *A*'s, where the correlation peaks could be small.

For scoring each pattern with respect to our description of *A*, the data that are required are the normalized correlation peak heights $S_i(x,y)$ and their location points (x_i, y_i). The scoring method involves setting up a "fuzzy fan" about each correlation point (x_i, y_i) on the correlation surface a_i of the pattern. The fans are extended to encompass every other correlation point (x_j, y_j), where $i \neq j$, as shown in Fig. 2. The points (x_j, y_j) on the correlation surfaces of the primitives could lie anywhere between the fan boundaries, or outside the fans specified, depending on the type of input character. Since three primitives were chosen for the character *A* we have a total of six fans, two from each correlation point.

We define m_{ij} as the fuzzy membership of the feature in the class of primitive a_j, in the cone enclosed by fans drawn from the primitive correlation point (x_i, y_i) on correlation surface a_i. For six fans we have six fuzzy membership values m_{ij}, $i, j = 1, 2, 3$, where $i \neq j$. m_{ij} is a function of the

fan angle q, which is determined by the correlation point (x_j, y_j). Mathematically we can express m_{ij} as:

$$\mu_{ij} = f(\theta) = f\left(x_j, y_j \middle| x_i, y_i\right). \quad (1)$$

Physically, it is the significance of the location of feature j at (x_j, y_j) in view of the assumed presence of the feature i at (x_i, y_i) in confirming the nature of the object.

Using the correlation peak values and the fuzzy memberships, we have tried four possible scoring methods which are:

$$N_1 = \sum_i \sum_j \mu_{ij} S_j, \quad i \neq j. \quad (2)$$

$$N_2 = \sum_i \prod_j \mu_{ij} S_j, \quad i \neq j. \quad (3)$$

$$N_3 = \sum_j \prod_i \mu_{ij} S_j, \quad i \neq j. \quad (4)$$

$$N_4 = \sum_i \prod_j \mu_{ij} S_i S_j, \quad i \neq j. \quad (5)$$

For this study i, j = 1,2,3.

The fan angles were determined by scanning several handprinted *A*'s and taking the average value of the angular variations of the different lines that form the character. The fan angles chosen for this study are shown in Table 1. For the fans drawn from the correlation point (x_1, y_1) on correlation surface a_1 to encompass the point (x_2, y_2) on correlation surface a_2, the total fan angle is 30°, with $q_1 = 15^\circ$ and $q_2 = 15^\circ$. The fuzzy membership m_{12} of the pattern in the primitive class a_2 is 1 along the bisector of the total angle, and

falls off to 0 linearly on either side. A similar set of angles and fuzzy membership values are specified for m_{13}. For the fans drawn from the correlation point (x_2, y_2) on correlation surface a_2 to encompass the point (x_3, y_3) on correlation surface a_3, $q_1 = 10°$ above the horizontal axis, and $q_2 = 5°$ below. The fuzzy membership m_{23} of the pattern in the primitive class a_3 is 1 along the horizontal axis and falls off to 0 on either side. A similar set of angles and fuzzy membership values are specified for m_{32} with $q_1 = 5°$ above the horizontal axis, and $q_2 = 10°$ below. For the fans drawn from the points (x_2, y_2) and (x_3, y_3), the fan angles and fuzzy membership values, m_{21} and m_{31}, are the same as those specified for m_{12} and m_{13}.

Fig. 3 shows the correlations obtained with the *A* primitives for a set of computer generated characters *A*, *A*, *M*, and *U*. The dark spots are the correlation peaks, which have been auto-scaled separately for each character. None of the fuzzy membership values of the two *A*'s were zero, as the correlation peaks were found within the fans specified. All the fuzzy membership values obtained for the computer generated character *M* were zero. The correlations obtained of this character with the three *A*- primitives violated our syntax description. Correlation peaks were obtained in a cluster as seen in Fig. 3. The more intense peaks of the three primitive correlations were found in the right branch cluster of the character. The correlation point obtained with primitive a_1 was located below and to the right of the

correlation point with primitive a_2, and was on the same level as the a_3 primitive correlation point. This does not fit our description of *A*.

For the computer generated *U*, correlation points obtained with primitives a_1 and a_2 were found in the left branch of the character, and were very close to each other. Though fuzzy membership values were obtained for m_{12}, and m_{21}, these were discarded taking into consideration the dimensions of our *A*'s. The correlation point with primitive a_3 was obtained in the right branch within the fans specified, and more or less on the same level as the correlation point with primitive a_2, which gave fuzzy membership values m_{23}, and m_{32}.

The normalized scores obtained for some of the test characters are shown in Table 2. The characters do not constitute a representative set but they are illustrative examples on which our scoring method was tested and proved. Good scores were obtained for the two computer generated *A*'s, A_1 and A_2. The primitives were formed from the character A_1 but a higher score was obtained for A_2, which gave a better correlation. For the handprinted A's, the scores were acceptable for A_3 and A_4. Our method fails on the basis of syntax, the specifications of which were stringent, for the character A_5. The method also failed to recognize the character A_6 on the basis of poor feature recognition. None of the features fit our template well. The scores obtained for all the *M*'s were zero except for one handprinted *M*, which is also shown in Table 2. The primitive correlations which were obtained in the left branch of the character, did not violate our syntax description. Scores were obtained

for the character as two of its correlation points, corresponding to correlation surfaces a_2 and a_3, were found to lie within the fan boundaries drawn from correlation point on surface a_1. On comparison with the *A* scores it can be seen that the scores are not as high as those obtained for the good *A*'s. The scores obtained for all the *U*'s except the computer generated one, were zero. From the table, it can be concluded that for patterns having some features in common with *A*, like the *M*, the scoring method "N_4" works best as it uses multiplicative features which are high only if all features are present and well-formed.

In summary, we have designed and implemented a new and improved scoring procedure for pattern recognition using the syntactic approach, which is successful in eliminating patterns that do not belong to the class under consideration. Our method can be used to recognize multiple objects in the scene that are non-overlapping.

This work was sponsored by the U. S. Army MICOM under contract no. DAAH01-93-C-R351 , and the U. S. Department of Energy under contract no. DEFG05-94ER25229.

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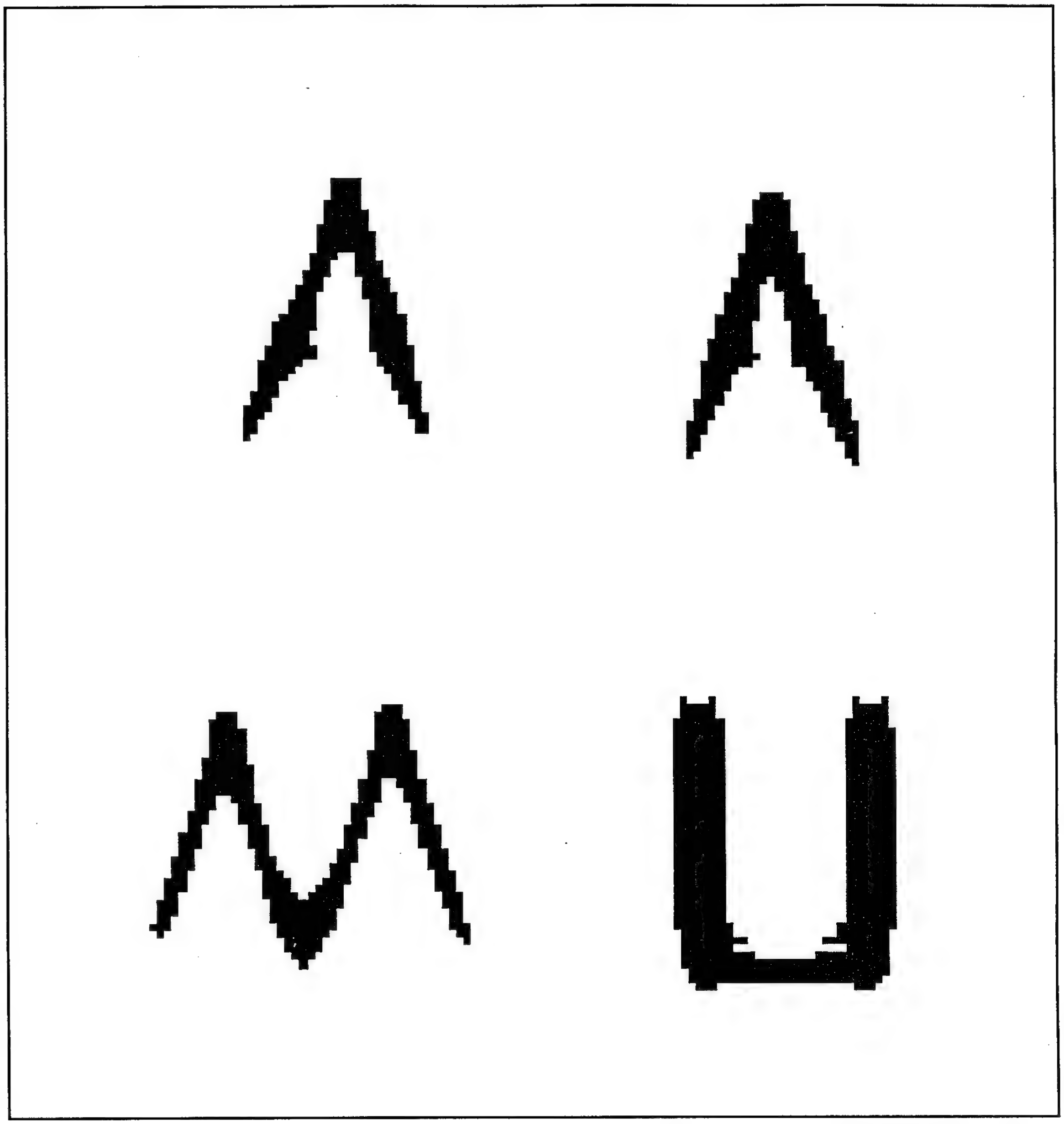
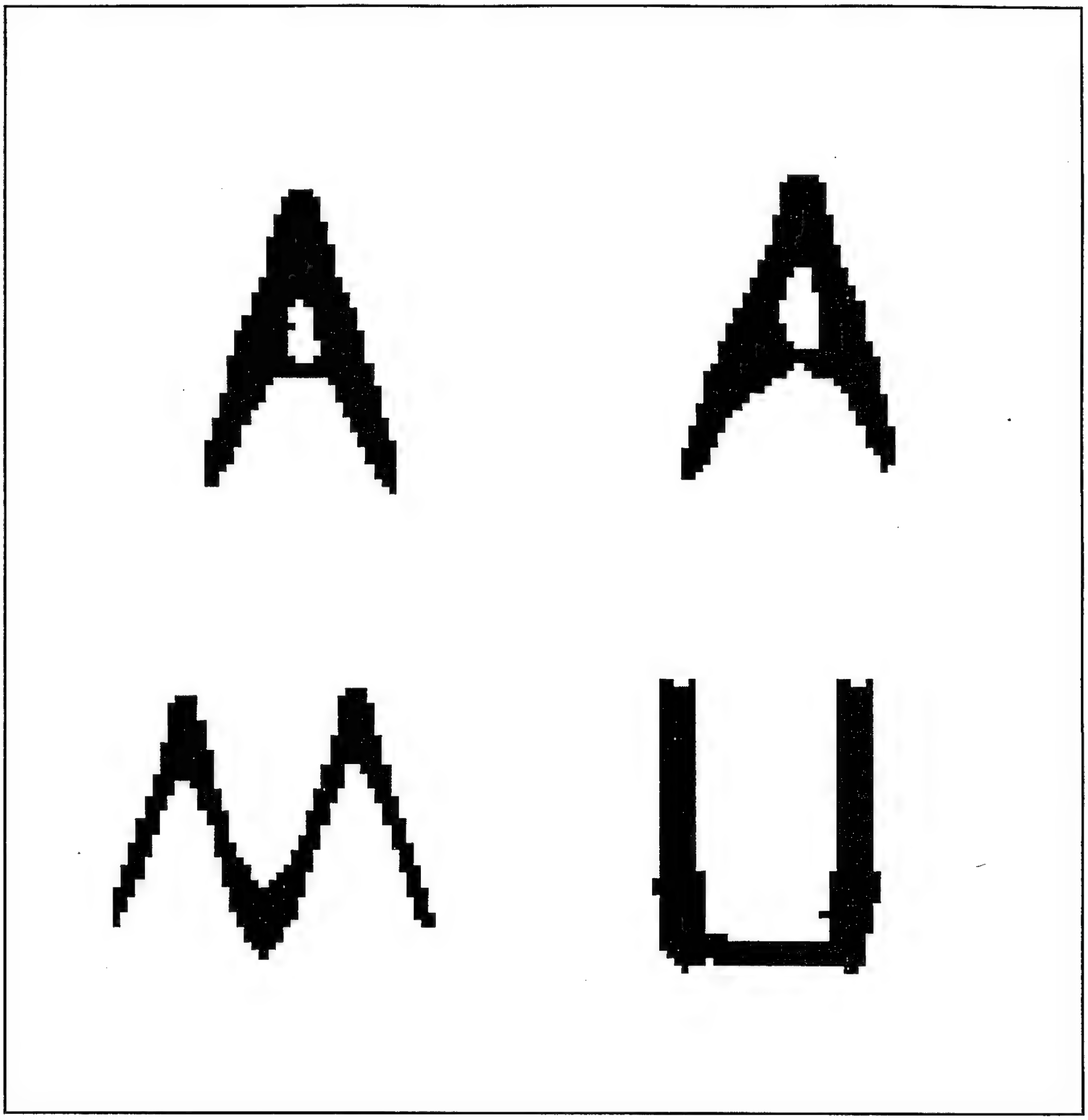
Figure Captions:

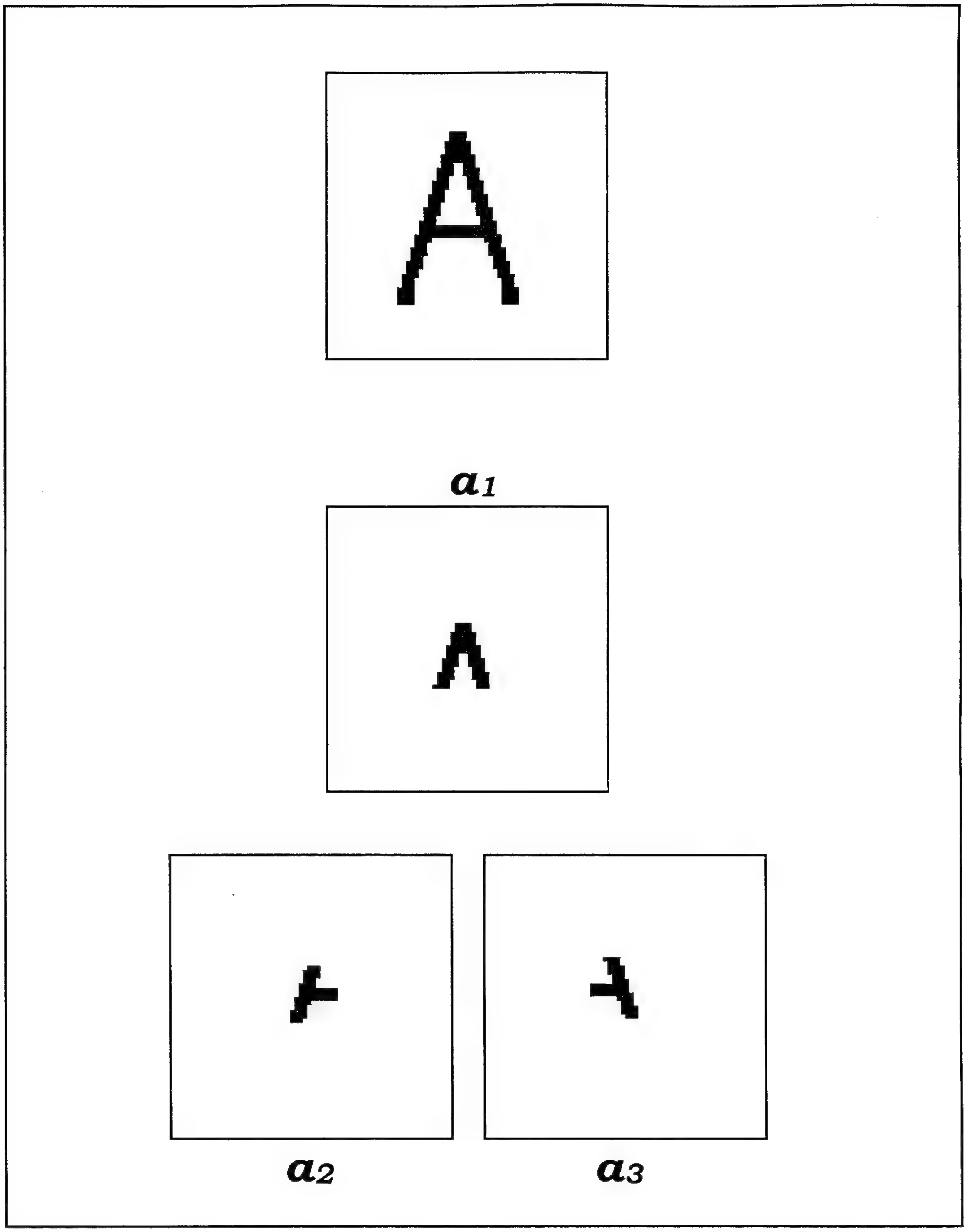
Fig. 1. Character *A* and pattern primitives a_1, a_2, and a_3.

Fig. 2. Fans drawn from correlation point (x_i, y_i) on correlation surface a_i to encompass correlation point (x_j, y_j) on correlation surface a_j. The circles represent the primitive correlation surfaces.

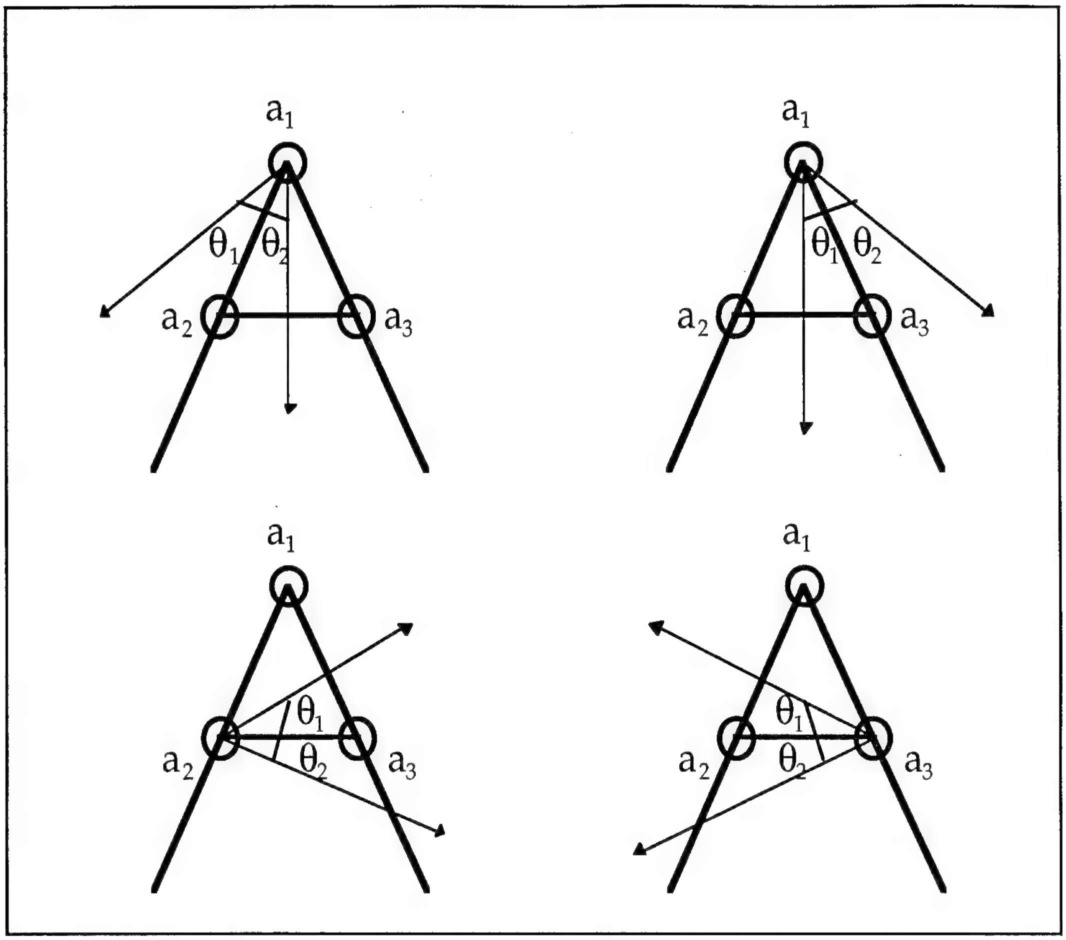
Fig. 3. Correlations obtained for a set of computer generated characters *A*, *A*, *M*, and *U*, with pattern primitives a_1, a_2, and a_3, which were derived from the top left character *A* shown in Figure. The dark spots are the correlation peaks.



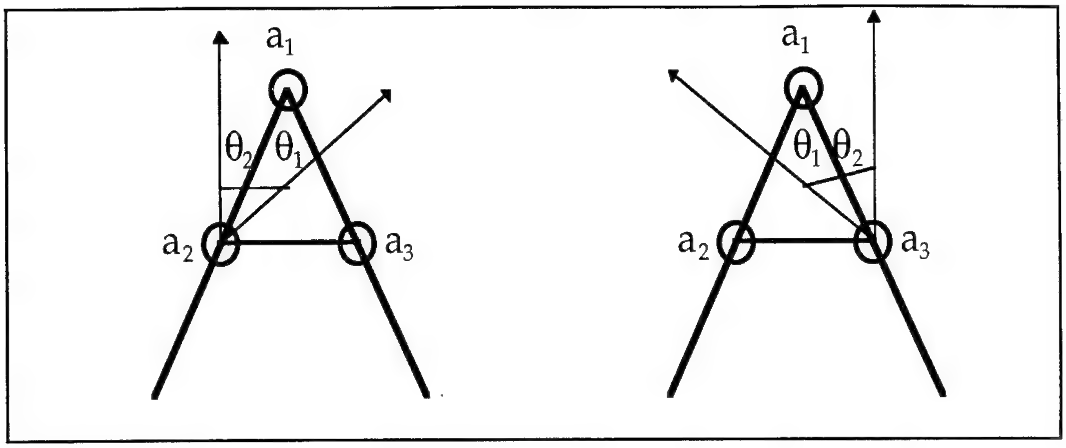

A A
M U



a_1
a_2
a_3



a_1
θ_1 θ_2
a_2 a_3
a_1
θ_1 θ_2
a_2 a_3
a_1
θ_1 θ_2
a_2 a_3
a_1
θ_1 θ_2
a_2 a_3



a1
θ2
θ1
a2
a3
a1
θ1
θ2
a2
a3

Table 1. Fan angles for fans drawn from each correlation point for determining fuzzy membership values.

Fans Drawn		Fan Angles (degrees)	
From (correlation surface)	To (correlation surface)	θ_1	θ_2
a_1	a_2	15	15
a_1	a_3	15	15
a_2	a_1	15	15
a_2	a_3	10	5
a_3	a_1	15	15
a_3	a_2	5	10

Fans Drawn		Fan Angles (degrees)	
From (correlation surface)	To (correlation surface)	θ_1	θ_2
a_1	a_2	15	15
a_1	a_3	15	15
a_2	a_1	10	5
a_2	a_3	5	10
a_3	a_1	15	15
a_3	a_2	15	15

a_2	a_3	10	5
a_3	a_1	15	15
a_3	a_2	5	10

Table 2. Normalized scores obtained for some test characters.

Normalized Scores	A1 (computer generated)	A2 (computer generated)	A3 (Hand printed)	A4 (Hand printed)	A5 (Hand printed)	A6 (Han printe
						
N_1	1	1.6	0.96	.67	0.009	.36
N_2	1	2.48	1.009	.46	0	0
N_3	1	2.42	1.21	.76	0	0
N_4	1	2.57	0.77	.29	0	0

One-, two-, and three-beam optical complexity effects in photorefractive materials (from Chaos to Logos)

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ABSTRACT

Collective selforganization effects and chaos are commonly observed in optics. We describe examples in a particular kind of nonlinear optical material: photrefractive crystals. In particular, we show different effects which arise when photorefractive crystals are illuminated by on laser beam, two laser beams, and three laser beams.
Key words: Photorefractives, Selforganization, Hexagonal Patterns, Laser, Phase Conjugation

I. INTRODUCTION

Classical optics is linear. It is complex technically. We are still learning how to characterize coherence, polarization, imaging, etc. "Complexity" as mathematicians use that term has become important as we introduce materials which interact non linearly with the incident light into optical systems and use optical systems which introduce feedback or other coupling. Inevitably, however, non linearity and coupling open the door to the two seemingly-opposed manifestations of complexity: the emergence of order and the emergence of chaos. Indeed there is an order to chaos so that apparent contradiction is illusory.

We welcome the selforganization of this journal as a forum wherein workers from many fields can interact in a nonlinear manner in the hope of giving birth to emergent order or chaos. Toward that end, we offer this invitation to dialog.

Complexity is ubiquitous in modern optics. We offer here not a survey but a sampling. There is much more than we show here, but what we show here is enough to paint an accurate abstract picture of complexity in one field of optics called nonlinear optics.

II. NONLINEAR OPTICS AND LASERS: THE EDGE OF CHAOS

For most practical purposes, light does not interact with light. Even what we call "interference" is best thought of as the linear superposition of multiple independent electromagnetic fields. The description of these "simple" effects is

coherence theory. Great scientists have spent distinguished careers in this field and much remains to be done.

Indeed much of both current and historic optics is linear. This includes interferometry, optical instrumentation(microscopes, telescopes), etc. Even more modern applications such as information, communication, and storage is mostly linear. We use "square law" detection or recording, but seldom exploit this nonlinearity creatively. The primary exception is heterodyne detection.

What is usually meant by nonlinear optics is the use of a material whose interaction with other light (from the same or a different source) through its index of refraction or absorption varies in some polynomial fashion with the electric field of the light.* Many of the effects achievable with nonlinear optics are not what readers of this journal mean by "complexity." They include light frequency doubling, modulation of one beam by another, and a borderline case: optical bistability.

In optical bistability, the transmission or reflection of some device or material is governed by the irradiance (power per unit area) of the incident beam. Below a threshold, the transmission is low. Above threshold, it is high. Hysteresis is normally present. The usual conditions of nonlinearity and feedback create this phenomenon which is replete with the usual stigmata of complex, self-organized transitions: critical slowing down and critical fluctuations.

The key to much modern optics is the laser. It uses nonlinear spectroscopic

**Light can be viewed as an electric and magnetic field pattern satisfying the wave equation. We usually shorten this to say that light is an electromagnetic wave*

effects and feedback (from the system construction) to create an emergent, "synergetic," "coherent" output.

Thus both nonlinear optics and lasers put optics at "the edge of chaos." Here we will show some other examples of nonlinear optics at and over that edge. We will move from one-beam to two-beam to three beam configurations.

III. SINGLE BEAM SELF ORGANIZING PHENOMENA IN NONLINEAR OPTICS

Perhaps the most characteristic complex phenomena are the new organizations that emerge as systems are operated far from equilibrium. The phenomena has been perhaps most effectively celebrated (albeit from different perspectives) by Prigogine (1) and by Haken (2). The latter has given this field a name, "synergetics," that he and his many followers use.

When a single beam of laser light is incident on a crystal of some photrefractive crystals more-or-less on axis, the crystal undergoes self organizing behavior dependent on the beam irradiance. Within the range of irradiances we were willing to risk (These crystals sometimes scatter at high irradiance values), there is the sequence of states we can observe beginning with the lowest irradiances:

- transparency (no diffraction),
- circular pattern (leading to a cone of diffracted light),
- hexagonal patterns as shown in Fig. 1(leading to hexagonal dot array diffracted light, and
- rotating hexagons (leading to a rotating hexagonal dot array of diffracted light).

All of these transitions exhibit the three universal features of self organized state changes:

- thresholds in the control parameter,
- critical slowing down, and
- critical fluctuation.

Thus there is little doubt that this is a textbook case of selforganization.

We will offer a limited theory of how this happens below. The purpose is to illustrate the proposition that selforganization is not mysterious even though we are sometimes too dull to anticipate it before we observe it.

The theory has to do with mixing of off-crystal-axis scattering from the faces of the crystal with Fabry-Perot enhances on-crystal-axis scattering. At far enough off-axis beam incidence, the on-axis effect dies away and hexagons do not appear. Self organization does appear, however. The crystal structure and its diffracted light pattern fall into space-time chaos. no steady state arises.

The basic experiment involves shining a laser beam on a $KNbO_3$ crystal (not just any $KNbO_3$ crystal and not just any angle, but some crystals cut and illuminated within a reasonable small range of near normal angles). The control parameter is beam irradiance H. Or, since the area is constant, the laser power. At low power, the beam shines right trough the crystal as through it were a clear piece of glass. After a threshold irradiance is passed, the light passing through the crystal suddenly diffracts into a conical pattern. Soon thereafter, as H is increased, it diffracts the light into a hexagonal pattern - Fig. 1(a). If we look back into the crystal itself, we also see hexagons - Fig. 1(b). Opticists know that the diffraction pattern is the Fourier transform of the scattered pattern, so Fig. 1 (b) is what we would expect give Fig. 1 (a) and conversely. At even higher irradiances the hexagons begin to rotate. Are there "higher" states still, perhaps chaotic? Probably, but we are reluctant to look. These crystals are expensive and tend to crack with too much irradiance.

It is easy to give a simple explanation for this attractive phenomenon. Like most simple explanations it is almost certainly in need of detailed modifications. Yet it gives a nice "feel" for what is happening. One of the possible explanation is Raleigh-Benard selforganization that occurs as a fluid tries to configure itself optimally to dissipate more and more heat. Patterns of material and heat flow between front and back surfaces become the most efficient approach at high enough pumping irradiances, when surface forces dominate the price paid to organize , the favored shape is a hexagonal. Indeed such hexagons occur not only in this case and in Rayleigh-Bernard cases but also in many other situations. This illustrates something we have observed in complex phenomena for which there either is no name or the name is unknown to us:

There are universal behavior patterns in complex phenomena which are domain independent, and hexagonal pattern formation is one case. Feigenbaum's universal description of the bifurcation path to chaos is another.

IV. ANALYSIS OF THE SELFORGANIZATION OF HEXAGONAL PATTERNS

The single incident beam is broken into multiple beams inside the crystal. We will consider a general enough scheme of 6-wave interaction in a photorefractive crystal. To be specific, we describe first transformation and self-organization of scattering from a one beam setup. In Fig. 2, the incident wave C_1 is reflected from the crystal surface, producing wave C_1. Due to scattering, additional waves appear, from which the crystals select waves $C_{3,4}$ propagating along normal to the crystal surfaces(Fabry-Perot modes). Interaction of these 4-waves produce

holographic gratings and additional waves C_5 (counter-propagating to C_2) and C_6 (counter propagating to input C1 wave, being phase-conjugate replica of C_1).

This type of one-beam initiated 6-wave phase-conjugation was first observed in photorefractive SBN crystal [3], and explained as self- organization of scattering due to recording of transmission gratings. In this paper we show, that reflection gratings alone can also lead for self-organization of scattering, which is the case of $KNbO_3$ crystals [4].

For better understanding of holographic wave coupling in selforganization, it is helpful to introduce scattering diagrams. These lead to the proper terms in coupled-wave equations [5]. For example, for the Fabry-Perot mode wave C_3 (the left hand side) yield scattering diagram 1. Wave vectors indicate scattering waves, solid vector designate recording waves, and parallel lines show the orientation of holographic gratings planes. The first three-scattering diagrams on the right-hand side describe self diffraction when the same waves participate in grating recording and diffraction. The last two diagrams describe parametric diffraction when two waves make the hologram, and diffraction of the third wave results in the scattered wave C_3. Interplay between selfdiffraction and parametric scattering results in the complex dynamic of wave interaction leading to pattern formation. As an example, for the photorefractive crystal $KNbO_3$, the main recording mechanism is a nonlocal "diffusion" response that results in energy exchange by self diffraction and diffraction intensity and phase changes.

In that model, pattern formation may be described also as an interaction between different modes of a nonlinear Fabry-Perot resonator.

These modes are coupled via selfdiffraction and diffraction leading to the observed phenomenon of pattern interchanges and rotation. Another practically important scheme, described by a six wave-mixing model, is the hexagonal configuration, in which one beam is along the normal to the surface (wave C_4 in Fig. 2), and another beam is the C_1 wave. In this case strong waves C_5 and phase-conjugate waves C_6 appear in addition to the hexagon formation in the C_3 and C_4 waves. This scheme also allows us to perform edge enhancement of an image, when the input field C_1 is the Fourier transform of an object, provided the object is slightly displaced from the focal plane of the fourier transform lens. For a better introduction of different contributions to the C_5 wave, consider scattering diagram (Fig. 4).

For the case when C_4 is a plane wave and an image is introduced in the C_1 wave, the dominant scattering diagram will be 4th, describing parametric diffraction. This diagram shows that C_5 wave is phase-conjugate to C_2, and because C_2 is simply a reflected copy of the signal wave C_1. So, in this case, we have simultaneously 2 phase-conjugated waves, one is a reflected C_6 and another is C_5 - a forward propagation wave.

Another special case, normal incidence of the pump wave C_1, is also described by this 6-wave scattering model. For the plane-wave approximation, hexagon formation can be described by the 3 sets of 6-wave interacts (Fig. 2), where side waves $C_{1,2,5,6}$ are the hexagon spots. In this "holographic" language, the set of holographic gratings, described by the scattering diagrams 1 and 2, are responsible for formation of near-field hexagonal pattern (Fig. 5).

Another, equivalent language of pattern formation in photorefractive crystal described the above-mentioned 6-wave scheme as interaction of Fabry-Perot resonator's modes. In both languages, holographic and resonator's coupled modes may be used for description of the spatial-temporal pattern formation and the choice of model depends on convenience and the background of interested researches.

Finally we will describe a simple variation of the experimental procedure described above to achieve broadcasting of an input object into more than one location in space. Traditionally, the decomposition of the optical beams into several directions can be achieved using gratings, or diffractive optics in the general case. To achieve image broadcasting, we reduce the angle between the two participating optical fields C_4 and C_1 in our case to zero. Furthermore, we first expose the crystal to the pump beam C_4 for a few second to set up the transmission and reflection gratings which are responsible for directing the pump into a hexagonal pattern in the far field. Thereafter, with the pump switched off, the crystal is illuminated with C_4, which is Fourier transform of the object.

The far-field pattern is shown in Fig. 6 and shows the broadcasting of the letter "T" to the locations of the hexagonal pattern. It is also possible to change the orientation of the pattern by exposing the crystal to the pump beam sufficiently long and so that the pump makes a small angle (approximately 0.04 degrees) with the normal to the incident surface of the crystal. The prolonged exposure reorients the far hexagonal pattern. Reillumination of the induced gratings created by the pump with the Fourier transform of the object broadcasts the images at this new orientation.

Besides "central" broadcasting to all the six locations of the hexagon, we expect that local broadcasting is also possible by redirecting the image into *one* of the peripheral spots. In this case, only three nearest neighbor spots will be communicated with, leaving another three spots free for another independent message.

V. TWO WAVE SELFORGANIZATION

When we go to two beams incident on a photorefractive* crystal, new complex phenomena emerge. One of our favorite of these is the Double Phase Conjguate Mirror or DPCM. Initial "noise holograms" formed by two independent, mutually-incoherent beams interact in the crystal to form a new hologram — one which causes each beam to retrace the path of the other. For obvious reasons, we sometimes refer to this effect as "light calling" [6,7].

VI. THREE BEAM SELFORGANIZATION

As interesting as the DPCM is, it only sets the stage for what happens when we add a third beam. As readers of this journal know well, three is a magic number in complexity. Three coupled nonlinear variables introduce the possibility of chaos. In this case, we see not only chaos, but also a beautiful and unusual path to chaos. We call this phenomenon a TPCM or a Triple Phase Conjugate Mirror.

Figure 7 shows a DPCM setup. To build a TPCM, we added a third beam on one side as shown in Figure 8. We then monitored the combined beam produced when both of the mutually coherent beams on one side were called into a single beam on the other. We used a chopper in conjuction with a mode locked amplifier to distinguish

point in time to destructively subtract, is not fully understood. We do know that for a photorefractive volume hologram the induced Bragg grating will readjust given the light intensity distribution. A DPCM, with two input beams, reaches a stable solution after some period of time. The TPCM, however, does not have such a point, but goes into an orbit about such a point.

The distressing thing about chaos is that it is not repeatable. No two runs of this experiment can be overlaid. The run we show in Figure 9 is as close to a "typical" time series as there ever was. For comparison, we show in Figure 10 what happens when one of the beams is blocked, giving an ordinary DPCM. Again there is a simple, largely-correct explanation. The TPCM couples two mutually-coherent beams into one output. When the two beams are in phase, the combined beam is bright. Of course, when they are out of phase, it is dark. The most interesting thing is that the conjugated beam loses its way of falls into a chaotic attractor, see Figure 11. The dimension of this attrator has been calculated to be between 0.7 and 0.8. The physical mechanism that causes the two beams to constructively add and at a later VI.

VII. CONCLUSION

Self organization in photorefractives is ubiquitous and almost inevitable. Our task is to make creative use of it, DPCM is obviously useful. But what about phenomena like hexagon formation? If our task is our task is to form hexagon, this is a wonderful way to do it. Generalizing from that leads to the systematic consideration of "synergetic manufacturing" which we explore elsewhere (8).

VIII. ACKNOWLEDGMENT

This work was supported by the U.S. Air Force Materials Laboratory Under and STTR subcontract to American Propylaea Corporation and by the U.S. Army Space and Strategic Defense Command.

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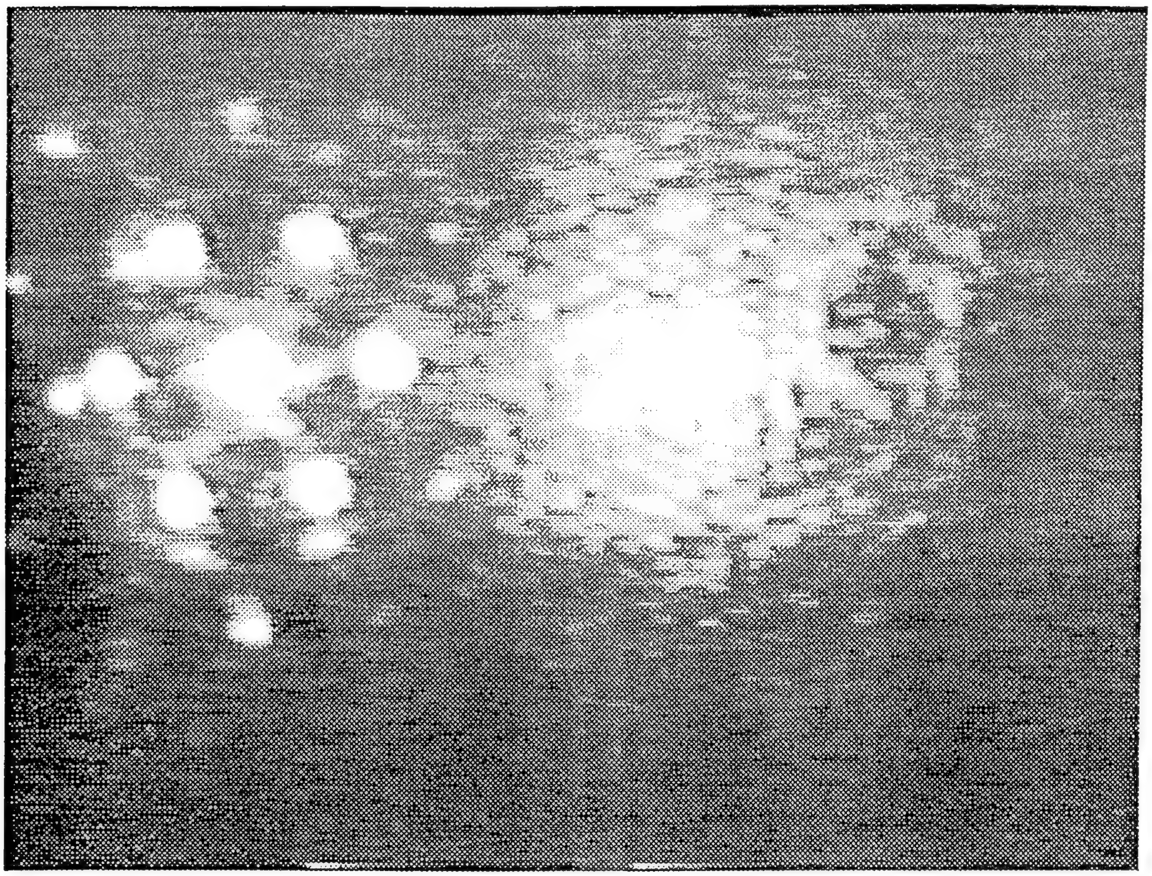

(a) (b)

Fig. 1. Far-field transmission pattern showing central spot and hexagonal spot array(left), near-field pattern showing hundreds of spots arranged in hexagonal arrays(right).

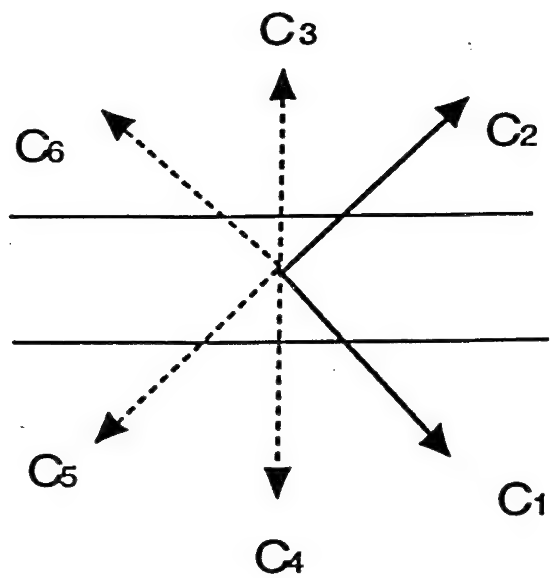

Fig.2 Scheme of 6-wave mixing generated by a single laser beam C1 (solid line vector). with reflected beam C_2 and scattering wave C_3=C_6

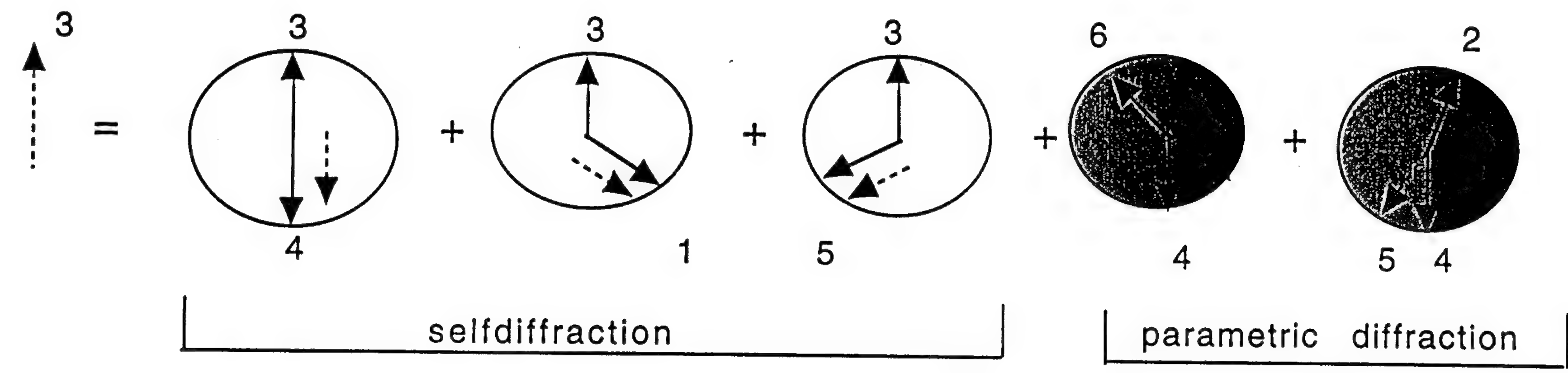

Fig. 3. Scattering for the Fabry-Perot mode wave

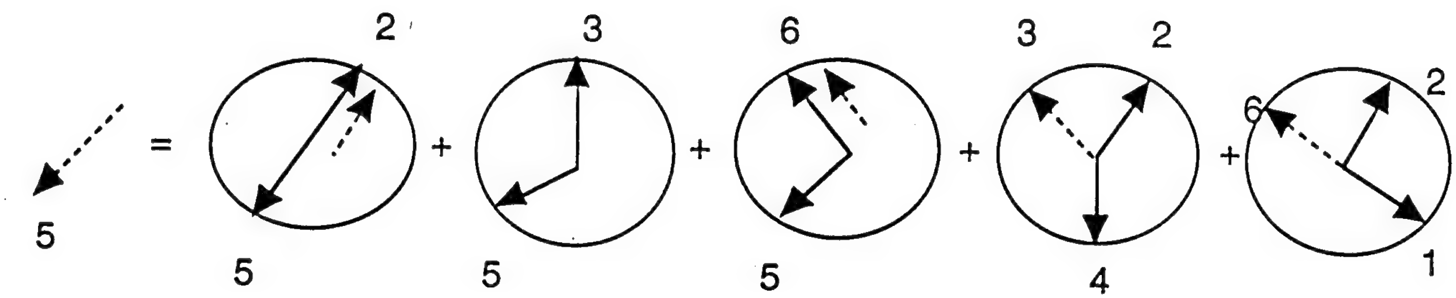

Fig.4. Scattering diagrams for C_5 waves

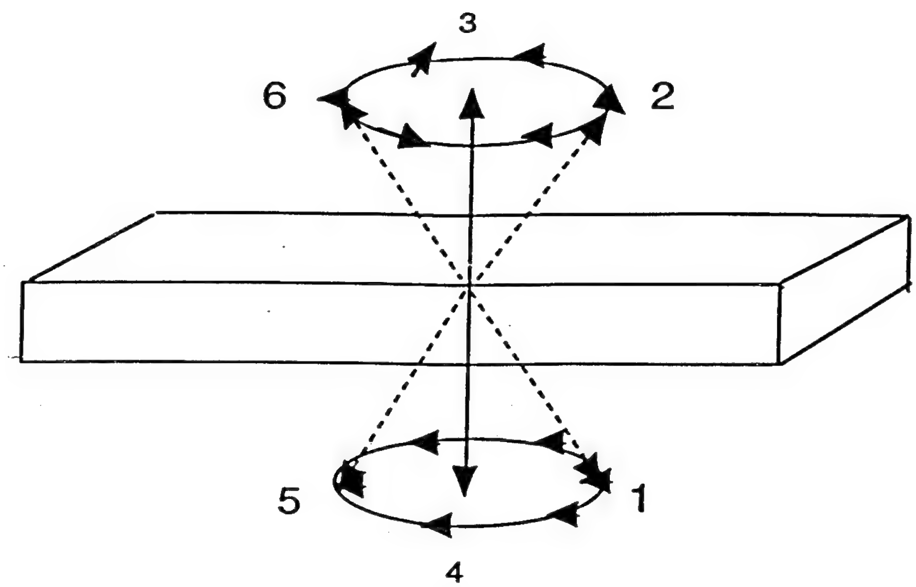

Fig.5. Holographic scheme of hexagon formation discribed by interaction of 14 plane waves.

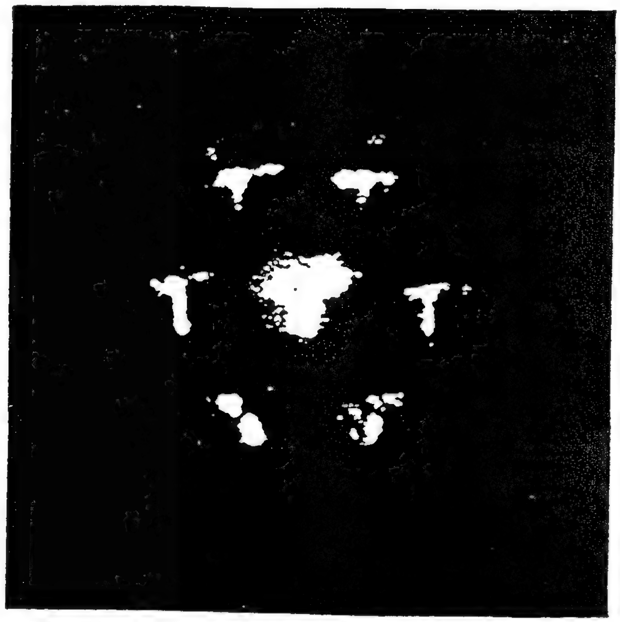

Figure 6. Letter "T" broadcasted to the locations of the hexagonal pattern.

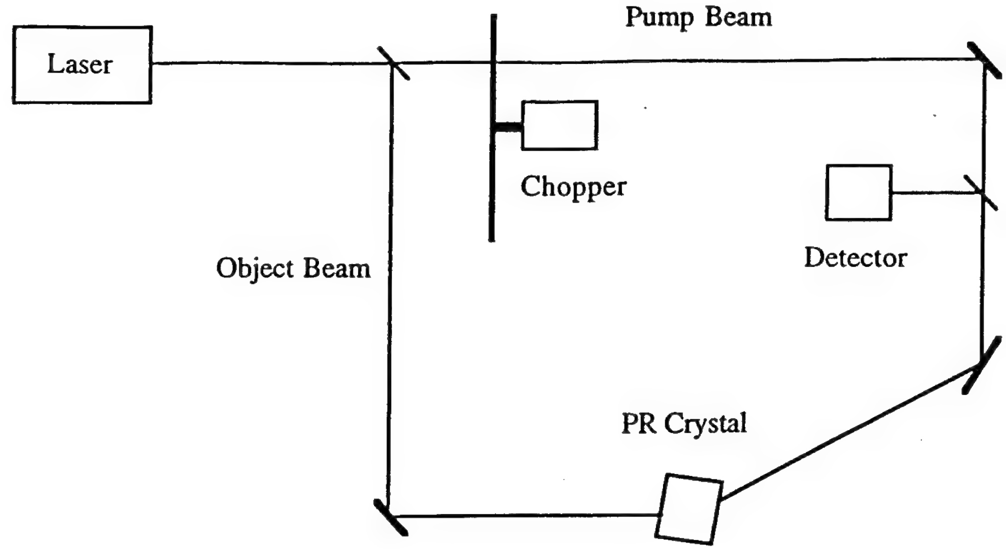

Figure 7. DPCM experimental setup.

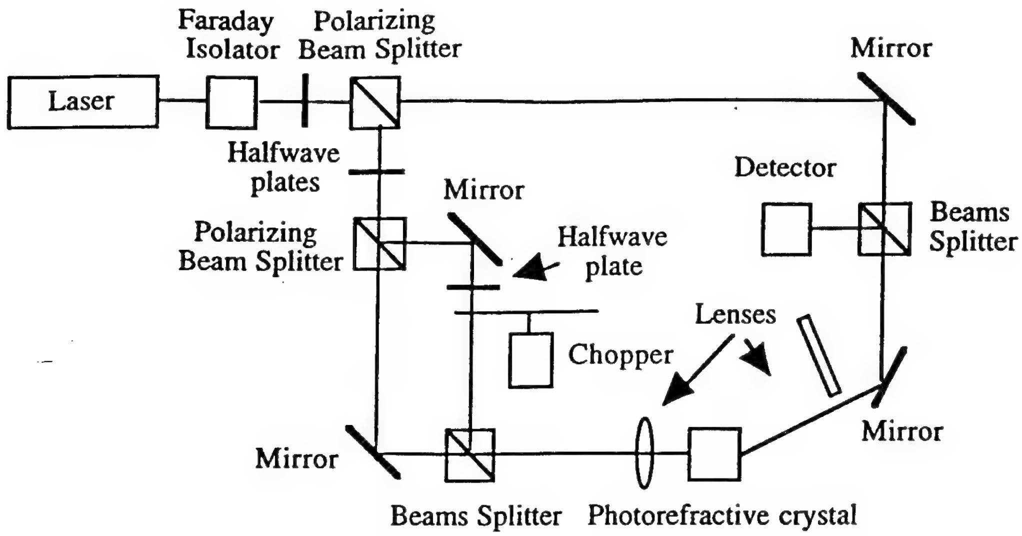

Figure 8. TPCM experimental setup.

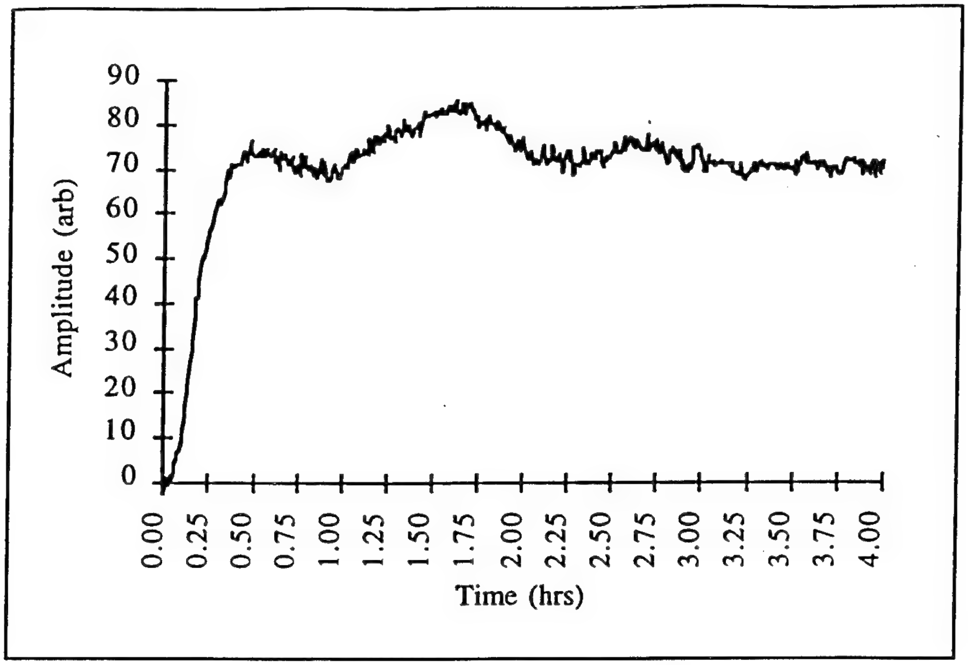

Figure 9. Typical DPDM time series.

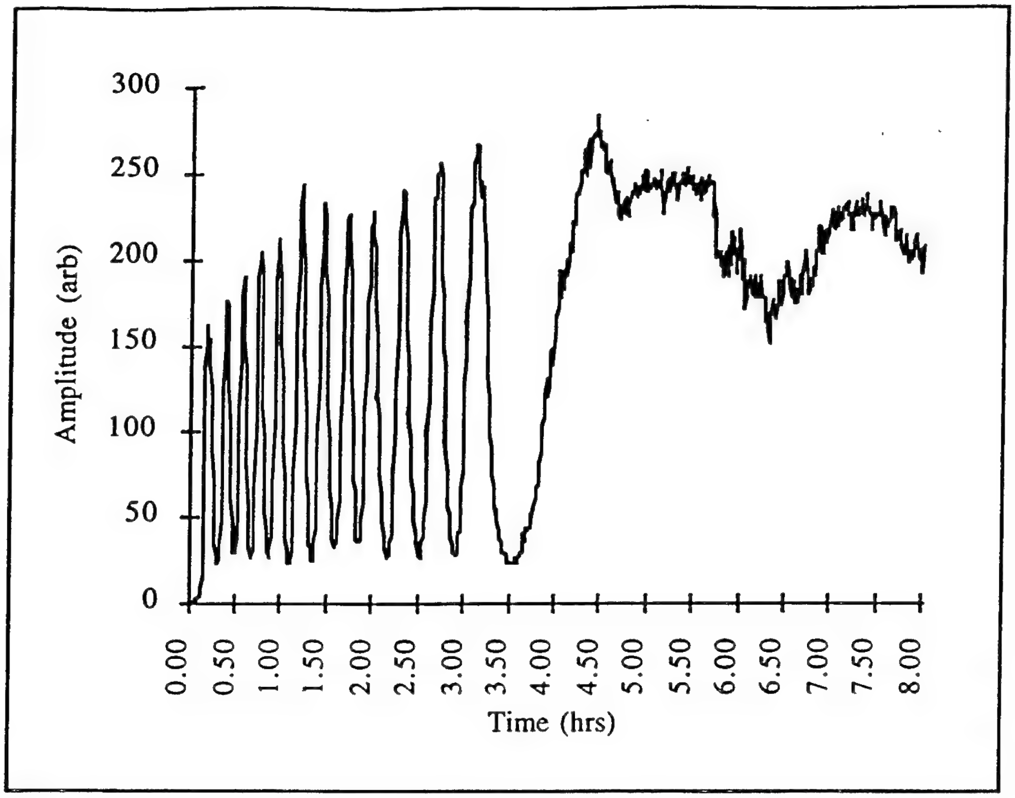

Figure 10. Typical TPCM time series.

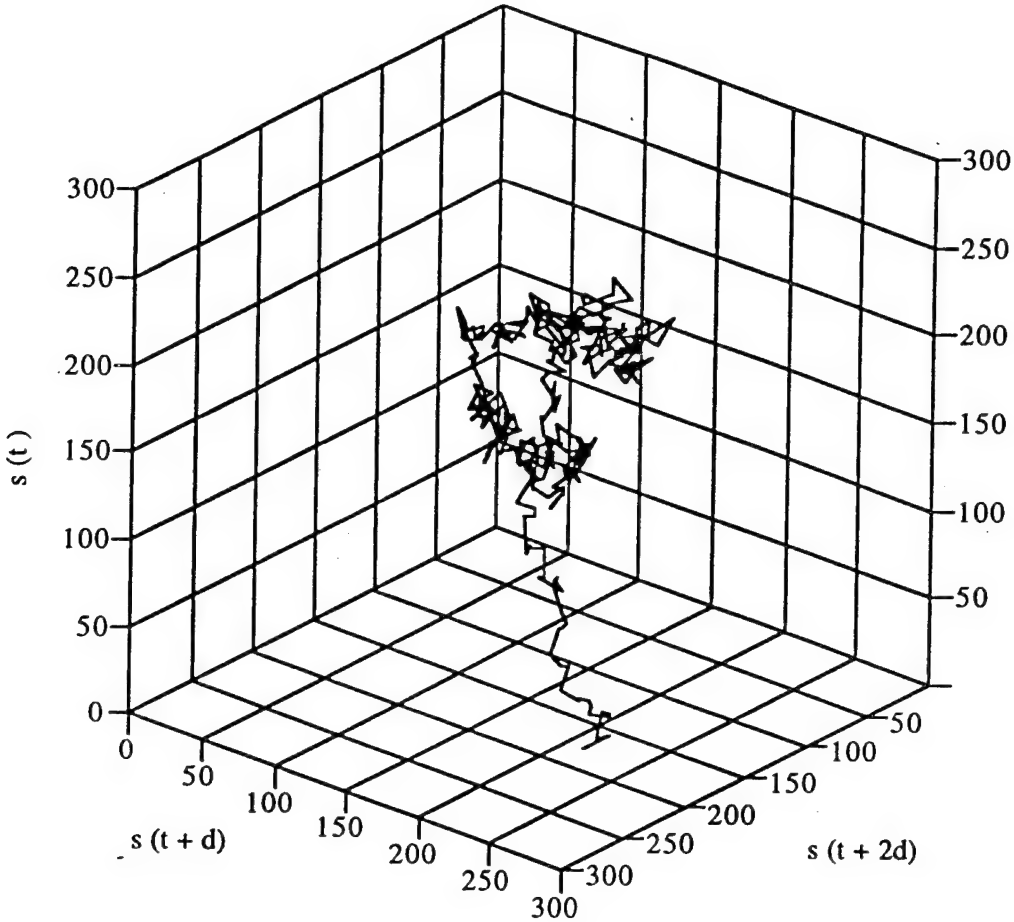

Figure 11. 3-D embeding of Figure 10.

Continuous Extensions in Optics

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Abstract

A continuous extension is an operator which replaces an integer value used as input, output, and/or parameter of another operator with a real number in such a way that when the real number is specialized to integer values, the two operators are identical.

I. Introduction and Background

The purpose of this communication is to place a number of issues of importance to optics such as fuzzy logic, fractals, fractional Fourier transforms, fractional calculus, filtering and partial coherence, into a simple unified framework. We call that framework "continuous extensions." In the following, we will define and illustrate three distinct forms of continuous extensions and note some combinations. These definitions are mathematical. We follow that with brief discussion of optical continuous extensions.

Boolean logic has been widely used in the recent decades as main means of transmission and processing of information. Various devices as computers were constructed to work with this logic format. Recently, it has been discovered that fuzzy logic may be a useful tool for control and other applications. In this approach the truth of a signal of a signals are not only 1 (yes) or 0 (no) but also anywhere in the zone between 0 and 1. A simple optical implementation was also obtained [1]. This fuzzy logic approach is an example for a continuous extensions of the Boolean logic.

Likewise, a continuous extension occurred in the field of dimensionality. The definition of the conventional dimensions (a point is one dimensional, a line is two dimensional, and a volume is three dimensional) was extended to shapes that have items repeated periodically while their sizes are being monotonously decreased (coined "fractals") [2,3]. Thus, instead of integer numbers, the dimensions now accept meaning also for real values, e.g. 2.23, when fractals are concerned.

The Fourier transform (FT) is an important tool for analyze systems and processing signals. Recently, a continuous extensions of this transform was suggested and coined the "fractional Fourier transform" (FRT) [4]-[7]. This transformation was also implemented optically. The FRT order may accept now any real value. When the order equals to 1 them the FRT becomes the FT. Note the FT is space invariant (the FT of a shifted objected equals to the FT of the reference object multiplied by a linear phase factor) the FRT is partially space invariant [8,9]. The amount of the space invariance

depends upon the fractional order (when the order is 1 the FRT is space invariant, when the order is zero the FRT is space variant).

Another important continuous extension that is related to the FRT, is the extension of the calculus into fractional calculus where a fractional derivation and integral are defined [10]. Using the property $F\left\{\frac{df(x)}{dx}\right\} = -jwF(w)$(where F is the FT operator and F(w) is the FT of f(x), the fractional p derivation of F (w) was defined as (-jw)p F(w).

Now, consider a continuous extensions example that is connected with a Fourier plane filtering. In the Fourier plane, one may write the filter function as

$$H(x) = \frac{F^*(x)}{|F(x)|^n} \tag{1}$$

where F(x) is the FT of the reference object. Different possible values of n may be chosen for different optimizing different criteria [11]-[13]. For n=o the filter obtained is the matched filter and it performs an optimization according to the signal-to-noise criterion. When=1 the filter obtained is the phase only matched filter and it optimizes the output plane according to the efficiency criterion (14). For n=2, the filter is an inverse filter and it optimizes the output plane according to correlation peak sharpness. The continuous extension of this case is to replace n with any real value.

In the early days of optics, illumination was considered to be coherent or incoherent. That is the coherence is 1 or zero. This is for both space and time domains. The continuous extension of those illumination types is the partially coherent illumination [15]. The degree of the coherency may be important for different applications (for instance in pattern recognition systems it is preferable if the output is spatially incoherent since then the speckle effect, which is typical for coherent light, does not exist-see Ref. [15}).

II. Continuous Extensions On Arguments

Suppose we have an operator O_I (n) which operates only on integers (some or all of them). An operator O_c (x)is a continuous

extension on the argument of $O_I(n)(n)$, if the allowable arguments of c are real, including the integer {n} and that

$$O_c(x = n) = O_I(N) \tag{2}$$

For example, the Gamma function $\Gamma(x)$ is a continuous extension on the argument of the factorial function

$$n! = n(n-1)(n-2)...(2) \tag{3}$$

That is real x's are accepted by $\Gamma(x)$ and

$$\Gamma(x = n) = n! \tag{4}$$

III. Continuous Extensions on the Operator Parameter

Suppose we have an operator $O^{(P)}(x)$ which may operate on reals, integers, complex numbers. The operator is parametrized by an integer parameter *p*. Then an operator $O^{(y)}(x)$ is a continuous extension on the parameter p if $O^{(y)}(x)$ is defined for real y which includes the allowable integer set {p} and

$$O^{(y = P)}(x) = O^{(P)}(x) \tag{5}$$

Immediately after inventing the differentiation operator

$$D^{(n)}\left[\int(x)\right] = \frac{d^{(n)}\int(x)}{dx^{(n)}} \tag{6}$$

Newton and Leibnitz independently invented fractional calculus [10].

Note that

$$D^{(0)}\left[\int(x)\right] = \int(x) \tag{7}$$

$$D^{(-1)}\left[\int(x)\right] = \int f(x)\,dx$$

Bracewell showed [16] the important of this extension for signal processing applications.

IV. Continuous Extension on Outputs

Suppose one has an operator that transforms x or or$\int(x)$ to n:

$$O(x) = N \tag{8}$$

That is, for all x O (x) is an integer. We call P(x) a continuous extension on the out put if

$$P(x) = y \tag{9}$$

where y is a continuous set of real numbers including the allowable integers {n} and

$$P(x) = O(x) \tag{10}$$

when y=n. The obvious example is the dimension operator. This operator gives

$$O(\text{point}) = O \tag{11}$$

$$O(\text{line}) = 1$$

$$O(\text{area}) = 2$$

$$O(\text{volume}) = 3$$

A continuous extension on the output for this case is the fractal dimension operator [2, 3].

V. Mixed continuous Extensions

Continuous extensions are possible on arguments, parameters and outputs. Either possible combination of digital/discrete choices is available in principle. Sometimes they are coupled. Since n! is an integer itself it is not surprising that $\Gamma(x)$ is a continuous extension on the argument and on the output. We could define a continuous extension on the argument of n! which retains an integer output. Let m=[x], where m is the highest integer such that $x \geq m$. Then ([x])!=m! is a continuous extension on the argument but not on the output.

VI. Conclusion

Table 1 summarizes some continuous extensions of interest in the optic field. This is a representative not an exhaustive list. Since a continuous extension cannot be less useful than the integer operator it contains, it is always worth investigating. In this communication we showed that each integer function can have several continuous extensions. The only example we provided was for n!, but there are also many others including fuzzy logic, many versions of fractional calculus, etc. Sometimes continuous extensions couples. We have shown recently, for example. that fractional Fourier transforms can implement several forms of partially space invariant correlation [17,18]. In this extension the fractal order varies and it is no longer uniform and discrete for the entire input plane. Using the concept and the mathematical analysis of the continuous extension, new optical system/operation may be invented quite deliberately. Each integer we encounter should be a challenge.

Acknowledgment

ZZ and DM acknowledge Adolf Lohmann's lecture "How to invent" that initialized their part of this work. H.JC"s work was supported by the U. S. Army Space and Strategic Defense Command under Contract Number DASG60-95-2-0001.

III. Locating Feature Primitives

Following an off-line selection process to pick the feature primitives, the location of the features is done optically. One of the several available methods can be used to design accommodating filters for each primitive in the Fourier plane. The auto-correlation of the derivatives of the researched primitive is characterized in the image plane by a sharp peak. A threshold value S_0 can be assigned to the normalized output signal S(x,y). The the output signal is given by:

With this condition, there will be either no peak or one peak will be obtained if we are dealing with single input objects. The data are the peaks and their centroids.

Syntactic pattern recognition can be used successfully to solve pattern recognition problems. We have used the syntactic approach to address a particular problem in pattern recognition. A new fuzzy relational scoring procedure for syntactic pattern recognition is illustrated which allows for robust recognition with tolerance to normal variations. This method is an application of conventional statistical methods of space invariant optical pattern recognition of features to syntactic pattern recognition of objects.

2.3 PARTIALLY SPACE VARIANT FILTERING

It is not strictly necessary that we break the two Fourier transform semiedempotent system into two equal parts. We can break it into unequal parts. One may do a fraction β of the task and the other may do the remaining 1-β fraction of the task. This attractive for two reason. First, we have the capability now to treat such systems analytically with "fractional fourier transforms." Second, this offers the potentially useful capability of partial space variance.

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III. COMPLEXITY

(See attached paper - "One-, two-, and three-beam optical complexity effects inphotrefractive materials (from Chaos to Logos)

IV. NONLINEAR OPTICS

Among the highlights in this field are three papers on channeling in photorefractives (13). This is a new diffraction condition (neither Ram-Nath nor Bragg) which is somewhat related to Talbot and Lau effects as well as to GRIN optics and may have some practical applications in focal plane arrays.

References

1. N. Kukhtarev, T. Kukhtareva, H.J. Caulfield, & A Knyazkov, "Two Dimensional Optical Channeling," Optik 97, 7-8 (1994).

V. LASERS/LASER MATERIALS

We have made significant advances in two areas: holographic probing of the properties of laser materials and powder lasers ( ). The latter may make solid state lasers cheap and rugged.

References

VI. FUNDAMENTALS

Two papers on fuzzy quantum mechanics (1,2) and one on quantum GA (3) were published. The former mark a new metaphysics many already find more satisfying than the mystery of complementary, the absurdity of many universes, etc. Along with recent work by Frieden and Soffer (4), they constitute the only derivations of the Schroedinger equation known to us.

The relationship between those two is being studied. This work is of great fundamental importances so Paper 2 is attached as an appendix.

References

1. H. John Caulfield, A. Granik and Luis Lopez, "A fuzzy logic metaphysics for quantum mechanics, Speculations in Science and Technology **18**, 61-67 (1995).

2. H. John Caulfield, A. Granik, "A quantum Mechanical Resolution of Multiple Metaphysical Paradoxes,"

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VII. CONCLUSIONS

In numerous areas we made both fundamental and applied contributions. At the same time we turned out three black Ph.D. opticists and proved that good scientists like

N. Kukhtarev, J. Shamir, B. Johnson, and J. Caulfield can do important work at an HBCU.

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Discrete Case	Continuous Extension	Extended Quantity	References
Boolean Logic	Fuzzy Logic	Argument and Output	1
Dimensions	Fractal Dimensions	Output	2,3
Fourier Transform	Fractional Fourier Transform	Parameter	4-7
Space Variance (or invariance)	Partial Space Variance (or invariance)	Parameter	8,9
Calculus	Fractional Calculus	Parameter	10
Fourier Plane Filter $\frac{F^*(x)}{\lvert F(x)\rvert^n}$ n=0 (matched) n=1 (phase only) n=2 (inverse)	Generalized Version $n \neq$ integer	Parameter	11-13
Coherency	Partial Coherence .	Parameter	15

Table 1: Important continuous extensions.

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A QUANTUM MECHANICAL RESOLUTION OF

MULTIPLE METAPHYSICAL PARADOXES

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ABSTRACT

We outline a series of metaphysical problems and paradoxes (being, life, mind, motion, locality and truth) which all have the same form. This fact allows us to reduce them to one seemingly simple problem of a transition from a "poor" reality to a "rich " reality. We made this reduction on the basis of fuzzy-logical metaphysics of quantum mechanics (Granik & Caulfield, Refs. 1,2). As a result we show that none of the metaphysical paradoxes is paradoxical if they are viewed from the point of view of the fuzzy-logical metaphysics.

I. INTRODUCTION

Physics and metaphysics are not separated by a sharp impenetrable boundary. On the contrary, one may say that the boundary (if it exists at all) is rather diffuse. Those who are trying to formulate a new physical theory, or pondering the epistemology or ontology of the existing theories, inevitably find (to a dismay of some, and to a satisfaction of the others) that they trespass the boundary between physics and metaphysics, and venture to the other Side. This is true for both pure philosophers of science (for example, Rauschenbach), and for pure physicists (Newton, Einstein, Bohr, Schroedinger, Heisenberg, and many others). We can safely say that not a single new theory in physics was not preceded by some metaphysical considerations. Inversely, any metaphysical study requires a serious study of a physical theory.

This explains how we, physicists, found ourselves in the domain of readers and contributors of this journal - metaphysics. We are not going to claim that in our everyday practice we encounter serious metaphysical problems. However our recent work on an interpretation of quantum mechanics inevitably leads us to the domain of metaphysics. Here we began to ponder about some related problems, which eventually lead us to an effort to propose our solutions to some metaphysical paradoxes. We are sure that our solutions will not be met with a universal approval. Exactly this thought compels us to present them to the judgment of people who spend their careers in solving these problems.

As we found in our work on a new interpretation of quantum mechanics, the above problems can be restated and reassessed from a point of view dictated by our interpretation. As we have already indicated, the boundary separating physics and metaphysics is rather fuzzy. Therefore our venture into the territory of metaphysics is inevitable since an interpretation of quantum mechanics necessarily leads one into metaphysical considerations. Thus this paper invites comments from the other side of the boundary hoping that they will enrich both.

II. PROBLEMS IN METAPHYSICS

In what follows we address some of the oldest and trickiest problems in metaphysics. The problems themselves are well known probably from the time immemorial. What may be new is how we formulate them by bringing forward their similarity which cannot be regarded as mere coincidence

Being. The problem associated with this concept is: How can being arise from nothing? This seems a major mystery. We hope to show that by resolving it we resolve all the other problems. We call this problem the Aristotle problem. The justification of the name lies in our reading of his proof of God's existence: There is no rational way to account for existence - therefore God exists.

Life. How can life arise from a non-living matter? We call this the Ezekiel's problem based on a famous quotation, "Can these bones live?" It is almost self-evident that this problem is similar to Aristotle problem.

Motion. How can motion arise from distinct, non-moving events, each occurring at a given instant of time? This is a well-known Zeno paradox that was quasi-solved by introducing calculus of infinitesimals. However this latter has its own metaphysical paradoxes.

Truth. How can truth arise from mere facts? In brief, what is the difference between truth and fact? While stories and parables almost universally convey the truth (or at least what is perceived as truth by their authors) science conveys only reproducible facts. It is not coincidental that religion and philosophy rest on truths not facts. Therefore honoring a famous teller of parables (or universal truths ?). We call this problem the Jesus problem: "What is truth?"

Non-locality. How can a measurement made at one point in space-time affect a second measurement at a space-time point so separated from the first one that any physical communications between two points is forbidden by relativity? This is a famous EPR paradox which in the opinion of its authors proved that quantum mechanics is flawed. In honor of the E of EPR (the most implacable critic of the current interpretation of quantum mechanics) we call this the Einstein problem.

This list could and most probably should be considerably extended. Our task is to reformulate the problems on the list. There emerges a common pattern in all these problems. There exist two kinds of reality "P" and "R "

(P) Is the reality that we provisionally call "poorer" and assume to be prior (both logically and temporally) to a reality which we provisionally call richer (R) . This means that here "poor" and rich" should be understood only in the context of a human perception. For example, "poor" nothing and "rich" being in the being problem means that nothing is viewed as a "poor" source which presumably cannot generate a "rich" variety of "things".

(R) is a seemingly "richer" reality which needs no ontological support from the prior "poorer" reality.

Thus all the above problems (or paradoxes) can be reduced to the following formal proposition:

$$P \Rightarrow R \qquad (1)$$

That is the logically prior "poor" reality P (i.e., non-being in the being problem) gives rise to the "rich" reality R. This violates common sense. In all other cases P $\Rightarrow$ R. Reading this equation from right to left produces what we ordinarily call a miracle. That living things (R) give rise to dead things is not surprising. The opposite is.

Implication (1) now requires an explanation of how one can account for the transition P $\Rightarrow$ R. Using this implication as a common feature of all the problems listed above we represent them in a table form - Table 1. As a justification for a such seemingly simple reduction of extremely complicated problems we rely on a well-known adage that a correct formulation of the problem constitutes half of its solution. Therefore we hope that our reduction given by relation (1) moves us along this path

TABLE 1

The P $\Rightarrow$ R Problem: How Does P Give Rise to P?

Name of a Problem	**Associated Person**	**Poorer, but Logically Prior Reality**	**Richer, but Ontologically Derivative Reality**
Being	Aristotle	Nothing	Everything
Life	Ezekiel	Matter	Life
Mind	Descartes	Brain	Mind
Motion	Zeno	Instants	Motion
Truth	Jesus	Facts, Observables	Deeper Truth
Locality	Einstein	Local, "Causal "Effects	Nonlocal, "Acausal" Effects

III. A FUZZY METAPHYSICS OF QUANTUM MECHANICS.

At the present, quantum mechanics is considered as one of the most profound physical theories. Its master equation, the Schroedinger equation, was preceded by some metaphysical considerations rooted in experimental results. In fact, in one of his 6 famous papers on quantum mechanics E.Schroedinger (Schroedinger,1978, 27) wrote about the wave equation that " It is not even decided that it must definitely of the second order. Only **striving for simplicity** (emphasis is ours) leads us to try this to begin with". One can see that a clearly stated metaphysical goal of simplicity superseded any other possible physical considerations. The same approach was also characteristic for another founder of quantum mechanics, Dirac who judged emerging theories by their mathematical beauty.

On the other hand, any new physical theory required a refinement of initial, sometimes rudimentary metaphysics, which lead to its formulation.This pattern is especially pronounced in the development of quantum mechanics where the arguments about its interpretation and the respective metaphysics started from the first steps of the new theory and are continuing to this day (see, for example an interesting book by Stapp, 1993)

Our previous work on a connection of the fuzzy logic and quantum mechanics (Granik & Caulfiled, Refs. 1,2)

followed the same pattern as was described in the preceding paragraphs. At the beginning we adopted a metaphysical notion that a perceptible world (that is accessible to our senses, understood broadly as including sophisticated detecting devices) is a result of defuzzification of the "thing in itself" understood as a fuzzy wholeness. In this context the defuzzification means an interaction of the parts of the wholeness that generates averaged quantities whose detection- registration presents itself as the perceptible world. This - allowed us to connect the quantum-mechanical measurement with the defuzzification procedure of the all-embracing wholeness - "thing in itself". Humans are a part of this process, and as such represent a result of defuzzification of this wholeness

Thus, in brief we introduced a metaphysics which posits the following propositions:

o There is a hidden all-embracing reality representing a fuzzy wholeness, where for example, there are no physical entities possessing sharply defined existence at a point in space-time. In other words, by using a concept of membership density (defined by these authors for the first time,Granik and Caulfield, Ref.2), we state that physical things, generally speaking, have a membership density which is not delta-function like.

o Measurement is defuzzification. In other words, an interaction of different parts of the fuzzy wholeness (charged with endless possibilities) results in creation of a crisp reality - physical objects having sharply defined membership density, and capable of communicating with each other by virtue of signals whose velocity at the level of crisp reality cannot exceed a certain limiting value - the speed of light. Inevitably, the defuzzification process is accompanied by a loss of information about the fuzzy wholeness. In a sense, we are bound to know only a part of the truth about the fuzzy thing in itself - the part which emerges as a result of defuzzification. Note that chances of generation of a specific defuzzified object out of the fuzzy wholeness are governed by the former's membership in the fuzzy wholeness. This is equivalent to the fuzzy logic treatment of *containment* of a superset A in a subset $B \subseteq A$ which determines the frequency of appearance of outcomes B in a set of n trials when $n \rightarrow \infty$ (See Kosko,1992, for details).

o Theory describing the fuzzy wholeness must be continuous judging by a continuous character of the wholeness itself. Here we are in direct agreement with E.Schroedinger (Schroedinger,1978,45) who wrote (once again operating within a framework of metaphysics) that wave mechanics " is a step from a classical point-mechanics towards <u>continuum- theories</u> " (underlined in the original),). Using these metaphysical assumptions we were able

1. To arrive at the conclusion that quanta represent a final stage (detection) of a defuzzification process (Granik &Caulfield, Refs.1,2). It is interesting to mention that some other authors view photons, for example, not as particles but as discrete perturbations in the electromagnetic field filling the entire Universe (Jones,1994,70)

2. To provide a more rigorous derivation of the governing equation for the fuzzy wholeness - the Schroedinger equation (Refs. 1 and 2).

To judge merits of a metaphysics one has to have some criterion. To this end we propose what we called metametaphysical criterion: "That metaphysics which posits less physics and predicts more physics is preferred". It seems that our metaphysics satisfies this principle. Definitely this statement can be questioned. However we would like to ask a reader to accept our metaphysics, and see how it would resolve all problems P $\Rightarrow$ R

IV. RESOLVING P $\Rightarrow$ R PROBLEMS

Our approach to P $\Rightarrow$ R problems based on section III is surprisingly simple, if somewhat unusual. In view what is said above, one can see that problems P$\Rightarrow$ R sound paradoxical since they ascribe to the concepts “poor” and “rich” almost absolute meaning. However in section I we already indicated that these names are provisional. More correctly, the identification of realities as “poor” and “rich”, and the latter as being derivative of the former, is wrong. We have switched the labels for what they ought to be.

Actually, the reality identified in the conventionally posed metaphysical problems as "rich" is in fact a result of the defuzzification (earlier metaphysics would have called it the "collapse of the wave function") of the fuzzy wholeness which was earlier called "poor" reality. This means that the measured (defuzzified) events comprising the body of the perceptible world are in fact terribly impoverished, dead residue of the rich fuzzy reality. We have always feel some ironic satisfaction in knowledge that great American artist J.J. Audubon used to shoot the birds and stuff them so he could be able to paint them. This can be viewed as another , rather crude form of defuzzification Thus, in essence, our metaphysics removes the paradox from P $\Rightarrow$ R problem by simply identifying correctly what is "rich" reality and what is "poor". What was viewed as paradoxical P $\Rightarrow$ R implications are in fact cases of mislabeled by prequantum metaphysicists.

The whole body of evidence (matter) studied by physics is dead, lifeless, impoverished remains of a fuzzy reality which is omnipresent (characterized by extremely complicated fuzzy density function whose domain of definition is infinite), carries a boundless energy, and contains possibilities of endless varieties of physical and mental events. Physicists call this "quantum vacuum", whose good description is given by T.Hey and P.Walters (Hey & Walters, 1989,130), “ Instead of a place where nothing happens, the 'empty' box should now be regarded as a 'bubbling' soup of virtual particle/antiparticle pairs”. We have to note that we **disagree with a notion of particles** at this level, since they appear only as a result of defuzzification. In our picture we have to say a "bubbling soup of fuzzy wholeness."

If one considers the fuzzy wholeness as an equivalent of quantum vacuum then how can it be that it is a source of all things, including us? The answer follows from our metaphysics. Things (what was previously called "rich" reality) are in fact impoverished, crisp, defuzzified realities. The background from which realities emerge is "no thing."

However it contains a possibility (or potentiality) of creating (generating) things. Therefore we can say that nothing (that is no-thing) is richer than all things, and definitely richer than the things that we measure.

One of the ways to describe this is to introduce an idea of a negative membership. The negative membership is unobservable. However it entails a possibility of generating positive membership (realized in the process of defuzzification as the chance of appearance of a certain physical entity) at the expense of annihilation of this negative membership. In a sense negative membership in the fuzzy wholeness serves as a warehouse of possible positive memberships.

V. CASE-BY-CASE P $\Rightarrow$ R RESOLUTION

Having established the primacy (and richness) of fuzzy wholeness ("no thing") we can successfully resolve all the paradoxes posed at the beginning of our paper.

o Being. By being we usually understand "things" in a broad sense of the word. It is tacitly accepted that things are the products of no-things. The paradox lies in semantics. We have always mislabeled the poorer reality (things) as a richer reality, and vice versa, the richer reality (no-things) as a poorer reality. Therefore it is enough to invert the implication and we get it right: No-thing $\Rightarrow$ Thing.

o Life. Life does not arise from a dead matter (as was stated in the original version of this problem). On the contrary, the " dead " (or more correctly ,deprived, impoverished) matter arises from the living (rich, endowed) no-thing (fuzzy wholeness) in a process of a "partial killing", so-to-speak, that is defuzzification.

o Mind. Mind does not arise from a mindless matter. The possibilities of all things including mental process are imbedded in the fuzzy wholeness. Therefore part of the infinite mind that we can ascribe to this wholeness is going to be recreated in living (and maybe non-living?) things in the process of defuzzification.

o Motion. The world of things, emerging as a result of defuzzification, could never change at points in space-time as was correctly noticed by Zeno. Zeno's conclusion that motion is impossible would be unconditionally true, if things (understood in terms of defuzzified entities) possessed ontological primacy. However because the unobservable fuzzy reality is in the process of eternal motion its product (the defuzzified things) can appear at different space-time points.

o Truth. Ultimate truth is a logical equivalent of the fuzzy wholeness. The latter can be captured totally by any process of defuzzification, and analogously the former can be captured by facts which are "defuzzified remains" of the truth. We can say that truth is

not equivalent to a set (even infinite) of facts. Facts arise from truth by defuzzification with it consequent loss of information.

o **Locality.** Until the moment of measurement (defuzzification) reality was fuzzy and global. Measurements destroy some of the fuzziness, thus reducing the global membership density function (which was not a delta-function) to a density function that is delta-function-like. This process establishes the locality of measured events and simultaneously *changing instantly* the whole fuzzy reality (where the restriction on the signal propagation is not valid any more - it is not measurable for the whole fuzzy reality). This removes the mystery of the EPR phenomenon.

VI. CONCLUSION

Our brief investigation (across the fuzzy border) of the classical metaphysical paradoxes allowed us to reduce them to a single problem of $P \Rightarrow R$ transition, which in turn follows from the ontological interpretation of quantum mechanics where primary-ontological entity is assumed to be fuzzy wholeness charged with endless possibilities of creating "things" via the process of defuzzification. Since the explanation of the problems $P \Rightarrow R$ on that basis seem almost self-evident we argue that our metaphysics is best suited to satisfy the criterion of meta(metaphysics) requiring the most "economical" metaphysics underlying physical theory.

Moreover, our discussion left us greatly impressed with philosophers of the Orient, and particularly with Lao-Tsu who often spoke of the "Tao." This does not mean that we derive our metaphysics from his philosophy but the coincidence of some statements is uncanny. If, for example, we would like to use his language then we could have identified Tao with the fuzzy wholeness, namely R, that is no-thing . In particular, he wrote "Each something is a celebration of the nothing that supports it." A "lossy" transition from the fuzzy world to the crisp world is equivalent in a sense to his poetic words "The Tao which can be spoken is not the true Tao".

One of us (HJC) remembers but can not find the reference for a quotation from a great German theologian , E.Brunner: " I have never believed in the creed of the church, and I never hope to do so, because I do not wish to commit idolatry". Concluding our paper we would like to say that although Ezekiel's bones and Audubon's birds will never live, but the no-thing from which they arose will never cease to thrive. The no-thing is the ultimate (R) from which all (P) ultimately derives. This observation does not remove the mystery of existence but it identifies it more properly.

ACKNOWLEDGMENT

. Part of A. G's work was supported by the travel fund from the university of the Pacific. H.J.C's work was supported by the Air Force of Scientific Research.

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FUZZINESS IN QUANTUM MECHANICS

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ABSTRACT

It is shown that quantum mechanics can be regarded as what one could call a "fuzzy" mechanics, that is, the mechanics whose underlying logic is not an Aristotelian binary logic of classical mechanics but rather the fuzzy logic. From this point of view, classical mechanics is a crisp limit of a more general quantum mechanics based on the fuzzy logic. Using such an approach, the Schroedinger equation is derived from the Hamilton-Jacobi equation.

A deep underlying unity of both equations is connected to the fact that a unique "crisp" trajectory of a classical particle is "selected " out of many-continuum paths according to the principle of least action. This can be interpreted as a consequence of an assumption that a classical particle "resides" in every path of a set of many-continuum paths that collapse to the single trajectory of an observed classical motion.

The wave function is treated as a quantity describing a deterministic entity possessing a fuzzy character. As a logical consequence of such an interpretation, the complementarity principle and wave-particle duality concept can be abandoned in favor of an idea of a fuzzy deterministic microobject.

Key words: Quantum mechanics, Schroedinger equation, Fuzzy logic, Optical computing.

"...a very strange idea has been introduced - the possibility of a photon being partly in each of two states of polarization..."

Dirac.[1]

INTRODUCTION

One of the purposes of this paper is to bring together several topics: fuzzy logic, quantum mechanics and quantum computing. In the process, we will extend our prior analysis of a fuzzy logic interpretation of quantum mechanics[2] by showing that the Schroedinger equation can be derived from the assumptions of the fuzziness underlying not only quantum mechanics but also classical mechanics. A simple way to define fuzziness informally was provided by B.Kosko[3], who wrote that the fuzzy principle states that everything is a matter of degree. More rigorously the fuzziness is defined as multivalence.

Interestingly enough, even separation of classical and quantum domains is somewhat fuzzy since there is no crisp boundary separating them (see, for example,Ref. 4). Moreover, we can even claim that the difference between these domains is only in a degree of fuzziness which in itself is a fuzzy concept. In fact, both classical and quantum mechanics make predictions based on repetitive measurements which implies a certain spread of results. The crisp character of the formal apparatus of classical mechanics masks this important fact by a seemingly absolute character of a single measurement. From this point of view the ultimate statements of classical mechanics are nothing more than the results of a certain averaging (defuzzification, meaning elimination of a spread) with a certain weight that could be called the *fuzziness density*. The difference between classical and quantum cases is in a degree of fuzziness which can be described formally by some function representing the fuzziness density. The latter varies from a "sharp" in the classical case to a "diffuse" in the quantum case.

If we assume that the "thing in itself " has a fuzzy and deterministic character, then it represents itself to the outside participants (not necessarily humans) as a random set, thus masking its deterministic essence. We argue that both classical and quantum mechanics stem from the same

fuzzy roots, and there is no sharp divide between them. They rather represent the fuzzy "thing in itself" in its different realizations. This also explains why some phenomena existing in a "strongly" fuzzy domain of quantum mechanics can not be realized in a "weakly" fuzzy domain of classical mechanics.

Thus if we accept quantum mechanics as a more general theory than classical mechanics, then it seems reasonable to expect that the former could be constructed independently from the latter. However the basic principles of quantum mechanics cannot be formulated even in principle without invoking some concepts of classical mechanics. Both theories share some basic common features, namely that they are rooted in the fuzzy reality. As a hindsight we may say that this fully justifies a belief expressed by H. Goldstein [(5)] that quantum mechanics is a repetition of classical mechanics suitably understood.

Our basic concept is that reality is fuzzy and nonlocal not only in space but also in time. In this sense idealized point-like particles of classical mechanics corresponding to the ultimate "sharpness" of the fuzziness density are non-existent. Any process of interaction (usually called measurement) between different parts of the fuzzy wholeness is viewed now as a continuous process of defuzzification. It forces a fuzzy reality into a crisp one. It is clear that the emerging crisp reality understood as a final step (we call it detection) of measurement carries less information that the underlying fuzzy reality. This means that there is an irreversible loss of information, conventionally identified as a "collapse of the wave function". Generally speaking, it is not a collapse but rather a "realization" of one of many possibilities existing within fuzzy reality. Any measurement (viewed as a process) rearranges this fuzzy reality anew, thus leading to different detection outcomes according to the changed fuzziness.

Therefore, it seems quite reasonable to expect that the classical theory cannot escape bearing some traces of the quantum theory which underlies it. In this light we would like to recall the words of P. Bridgman who remarked that the seeds and the sources of the ineptness of our thinking in the microscopic range are already contained in our present thinking applied to the large-

scale region. We should have been capable of discovery of the laws of the former by sufficiently acute analysis of our ordinary common sense thinking.

As we have already said, both classical and quantum mechanics should be viewed as statistical theories (cf. Ref. 6) with respect to an ensemble of repetitive experiments where each experiment must be carried out under identical conditions. The latter is a very restrictive statement stemming from the crisp-logical world view, and therefore not realizable even in a more general theoretical setting of the fuzzy reality. If we assume a fuzzy nature of "things" then statistical character of physical phenomena would follow not from their intrinsic randomness but from their fuzzy-deterministic nature which in turn expresses itself as randomness. Clearly, this definition of the statistical nature of both classical and quantum mechanics is applicable even to experiments with one particle.

Let us elaborate on this. Conventionally, a statistical theory is tied to "randomness". However, recent results in the theory of fuzzy logic have provided a deterministic definition for the relative-frequency count of identical outcomes (the probability of the outcome in the language of probability theorists) by expressing them as a measure of subsethood S (A,B), that is a degree to which set A is a subset of B [7]. To make it clear, suppose B contains N trials, and A contains N_A successful trials. Then $S(A,B) = N_A/N$. We would like to extend this concept to experimental outcomes of measurements performed on a classical particle. This would be possible if we would consider the classical particle as being simultaneously in all possible paths connecting two spatial points. In a nutshell, this approach coincides with the idea (expressed in different words) underlying the principle of least action.

To adapt the concept of fuzziness to a spatial localization of a particle, we introduce the notion of a membership in the spatial interval (1-, 2- or 3-D) which, generally speaking, would vary from one interval to another. The membership can be defined as follows. Let us say that we perform N experiments aimed at detecting a particle in a certain interval and find the particle in this interval N_A times. The membership of the particle in the interval is then defined as N_A/N. Formally the membership can be described with the help of the sigma function of Zadeh [8]. In turn,

this approach allows us to formally introduce the *membership density,* defined as the derivative of the membership function. If we denote the membership density by μ, then a degree of membership of a particle in an elemental volume ΔV is $\mu\Delta V$. According to this definition, the particle has a **zero** membership in a space interval of measure 0, that is, at a point. Such an apparently paradoxical conclusion indicates that in general we should base our estimation of fuzziness on the relative degree of membership instead of the absolute degree of membership.

In other words, given a degree of membership $\mu(x_i)$ dV of a particle in a volume dV containing x_i and a degree of membership $\mu(x_j)$ dV of this particle in a volume dV containing x_j we find *relative degree* of membership of the particle in both volumes: $\mu(x_i)/\mu(x_j)$. This expression represents also the relative degree of membership of the particle in the two points x_i and x_j, despite of the fact that its absolute degree of membership in these points is 0.

The importance of the relative degree of membership is due to the fact that experimentally the location of the particle is evaluated on the basis of its detection at a certain location in N_i experimental trials out of their total number N. As was shown by Kosko[7], the ratio N_i/N measures the degree to which a sample space of all elementary outcomes of experiments is a subset of a space of the successful outcomes, or in other words, a degree of membership of the sample space in the space of the successful outcomes. Therefore, in our case, the relative degree of membership $\mu(x_i)/\mu(xj)$ can be identified as the relative count of the successful outcomes (in a series of measurements) of finding the particle at points x_i and x_j.

In view of these definitions, the classical mechanical sigma curve of membership in a spatial interval is nothing more than a step-function. This simply means that up to a certain spatial point x, the degree of membership of a particle in an interval from say $-\infty$ to x is zero, and for any point y>x the degree of membership in any interval $(-\infty, y]$ is 1. The corresponding membership density is the delta-function. Thus the idealized picture of mechanical phenomena (particles occupying intervals of measure zero) indicates that they are <u>strictly</u> non- fuzzy, and are governed by a bivalent logic.

In reality, any physical "particle" occupies a small but nonzero spatial interval. This means that the membership density is a sharp function to a minimum fuzziness (as compared to the *absolute* minimum represented by the delta-function corresponding to a point-like particle).

On the other hand, in the microworld the fuzziness is maximal. In fact, if we accept the idea that a quantum-mechanical "particle" (which we will call a **microobject**) "resides" in different elemental volumes dV of a three-dimensional space with the varying degrees of residence (membership), then we can apply to such a microobject our concept of the membership density. In general, this density cannot be made arbitrarily narrow as is the case for a classical particle. The latter can be considered as the limiting case of the former when the membership density becomes delta-function-like. Moreover, the fuzziness in the microworld is even more subtle since mathematically it is described with the help of the complex-valued functions. This results in the emergence of the interference phenomenon for microobjects, which in the classical domain is an exclusive property of the waves and not particles. Therefore, mutually exclusive concepts of particles and waves in classical mechanics become inapplicable in the realm of fuzzy reality where "particles" and "waves" are not mutually exclusive concepts, but rather various expressions of fuzziness.

For example, the double slit experiment can be interpreted now as a microobject's "interference with itself" since it has a simultaneous membership in all the space including elemental volumes containing both slits. Since the total membership of a microobject in a given finite volume is fixed, any change in membership in one of the slits affects another thus leading to the interference effect. In the following, we "recover" the fuzziness of the quantum world by deriving the Schroedinger equation from the Hamilton-Jacobi equation, where the latter can be viewed as the result of the collapsed fuzziness of the quantum world.

DERIVATION OF THE SCHROEDINGER EQUATION

First, we show how the Hamilton-Jacobi equation for a classical particle in a conservative field can be derived from the second law of Newton, thus connecting it to the destroyed fuzziness.

A particle's motion between two fixed points, A and B, can in principle occur along any conceivable path (a "fuzzy" ensemble in a sense that a particle has membership in each of them) connecting these two points. In the observable reality, these paths "collapse" onto one observable path. Mathematically, this reduction is achieved by imposing a certain restriction on a certain global quantity (the action S), defined on the above family of paths.

To see that more clearly, let us consider the second law of Newton, and assume that there are many trajectories comprising a continuous set. This means in particular that the classical velocity is now a function of both the time and space coordinates $\vec{v} = \vec{v}(\vec{r}, t)$. Under this assumption we fix time t=T. Then (since the correspondence $\vec{r}$ to t is many to many) $\vec{r}$ is not fixed as was the case for a single trajectory, and the velocity would vary with $\vec{r}$. This is equivalent to considering points on different trajectories at the same time.

Our assumption means that the time derivative is now

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V} \bullet \nabla . \tag{1}$$

Having this in mind, we apply the *curl* operation to the second law of Newton for a single particle. Performing elementary vector operations we obtain

$$\frac{\partial}{\partial t} \text{curl } \vec{p} - \frac{1}{m} \text{curl } (\vec{p} \text{ x curl } \vec{p}) \equiv 0 \tag{2}$$

where $\vec{p} = m\vec{v}$ is the particle's momentum. If we view (2) as the equations with respect to curl $\vec{p}$, then one of its solutions is

$$\vec{p} = \vec{\nabla} S \tag{3}$$

where S($\vec{r}$,t) is some scalar function to be found.

Note that the spatial and time variables enter into S on equal footing. Therefore we can argue that S can serve as a function incorporating the notion of fuzziness (here a continuum of possible paths). Upon substitution of (2) back in the second law of Newton, $d\vec{p}/dt = - \nabla V$, where d/dt is understood in the above sense,

$$\nabla\ [\partial S/\partial t + (1/2m)(\ \nabla S)^2\ + V\ ] = 0$$

Integrating this equation and incorporating the constant of integration (which generally speaking is some function of time) into the function S, we arrive at the determining equation for the function S which is the familiar Hamilton-Jacobi equation for a classical particle in a potential field.

$$[\partial S/\partial t + (1/2m)(\ \nabla S)^2\ + V\ ] = 0 \tag{4}$$

By using Eq. (1) and (3) we can represent S as a functional defined on the continuum of paths connecting two given points, "0" and "1", corresponding to the moments of time, t_0 and t_1. To this end we rewrite (4).

$$\partial S/\partial t = p^2/2m - V \tag{5}$$

Integrating (5) we obtain the explicit expression of S in the form of the following functional

$$S = \int_{t_0}^{t_1} (\frac{p^2}{2m} - V)dt \tag{6}$$

which is the well-known *a priori* definition of the action for a particle moving in the potential field V. Thus we have connected the concept of fuzziness in classical mechanics with the action S. If we consider S as a measure of fuzziness in accordance with our previous discussion then by minimizing this functional (that is by postulating the principle of least action), we "eliminate" (or rather minimize) fuzziness by generating the unique trajectory of a classical particle. In a certain sense the principle of least action serves as a defuzzification procedure.

Now we proceed with the derivation of the Schroedinger equation. There are two basic experimental facts which make microobjects so different from classical particles. First, all the microscale phenomena are linear. Second, (which is corollary of the first) these phenomena obey the superposition principle. Here it is necessary to recall that already at the initial stages of development of quantum mechanics, Dirac formulated its fuzzy character, albeit without using the modern-day terminology. He wrote "... whenever the system is definitely in one state we can consider it as being partly in each of two or more other states" [1]. This is as close as one can come to the concepts of fuzzy sets and subsethood [9] without directly formulating them. In view of these concepts it does not seem strange that a microobject sometimes can exhibit wave properties. On the contrary, they arise quite naturally as soon as we accept the fuzzy basis (meaning "being partly in ... other states") of microscale phenomena which implies among other things the above-mentioned "self-interference."

How can we derive the equation that would incorporate these essential features of microscale phenomena and, under certain conditions, would yield the Hamilton-Jacobi equation of classical mechanics? We depart from the Hamilton-Jacobi equation (but not from the second law of Newton) because of its connection to the hidden fuzziness in classical mechanics. We consider the simplest classical object which would allow us to get the desired results that is to account for the two experimental fact mentioned earlier. Quite naturally, we choose a free particle which implies setting $V = 0$ in Equation 4. Our problem is somewhat simplified now. We are looking for a linear equation whose wave-like solution is simultaneously a solution of the Hamilton-Jacobi equation. Since the mechanical phenomena behave differently at micro– and macroscales the linear equation should contain a *scale factor* (that is to be scale-dependent), such that in the limiting case corresponding to the macroscopical value of this factor, we get the nonlinear Hamilton-Jacobi equation for a free particle.

A nonlinear equation admits a wave-like solution (for a complex wave) if this equation is homogeneous of order two. Since equation (4) does not satisfy this criterion, we cannot expect to

find a wave solution for the function S. However this turns out to be a blessing in disguise, because by employing a new variable in place of the action S, we can:

a) convert this equation into a homogeneous (of order two) equation (thus allowing for a wave-like solution) and

b) simultaneously introduce the scaling factor. It is easy to show that there is one and only one transformation of variables which would satisfy conditions a) and b):

$$S = K\ln\Psi \tag{7}$$

where the scaling factor K is to be found later.

Upon substitution of (7) in (4), we obtain the following homogeneous equation of order two with respect to the new function Ψ

$$K\Psi\frac{\partial\Psi}{\partial t} + \frac{K^2}{2m}(\nabla\Psi)^2 = 0 \tag{8}$$

Equation (8) is easily solved by the separation of variables, yielding

$$\Psi = C[-\frac{at - \sqrt{\frac{2m}{a}}\bar{a}\bullet\bar{r}}{K}] \tag{9}$$

where vector $\vec{a}$ of length a is another constant of integration. Since solution (9) must be a complex wave, the argument of Ψ must satisfy two conditions: a) it must be imaginary, and b) the factors at the variables t and $\vec{r}$ must be the frequency $\omega = 2\pi\nu$ and the wave vector $\vec{k}$ respectively. This results in he following:

$$K = -iB \tag{10}$$

and

$$a/B = \omega, \quad \sqrt{2m/a}\ \vec{a}/B = \vec{k} \tag{11}$$

where B is a real-valued constant. Now the solution (9) is

$$\Psi = C \exp [-i(\omega t - \vec{k}.\vec{r}] \tag{12}$$

Since both functions S and Ψ are related by Eq. (7), we can easily establish the connection between the kinematics parameters of the particle and the respective parameters ω and $\vec{k}$, which determine the wave-like solution of the Hamilton-Jacobi equation for the new variable Ψ. According to classical mechanics, $-\partial S / \partial t$ is the particle energy E_0 and ∇S is the particle momentum $\vec{p}$. On the other hand, these quantities can be expressed in terms of the new variable Ψ with the help of Eqs. (7) and (12), yielding the following:

$$E_o = B\omega, \qquad B\vec{k} = \vec{p}$$

From these relations we see that for a free particle its energy (momentum) is proportional to the frequency (wave vector) of the wave solution to the "scale-sensitive" modification of the Hamilton-Jacobi equation. The constant B is found by invoking the experimental fact that $E_0 = h\nu = \hbar\,\omega$ (where h is the Planck constant). This implies $B = \hbar$ or $K = -i\hbar$, and as a byproduct, the de Broglie equation $\vec{p} = \hbar\vec{k}$. Inserting solution (12) in original nonlinear equation (8), we arrive at the dispersion relation

$$\omega = \frac{\hbar}{2m} k^2 \tag{13}$$

Now we can find the linear wave equation whose solution and dispersion relation are given by Eqs. (12) and (13) respectively. Using an elementary vector identity,we rewrite Eq. (8)

$$[\frac{\partial}{\partial t} - \frac{i\hbar}{2m} \nabla^2] \Psi - \frac{i\hbar}{2m\Psi} [\operatorname{div}(\Psi \nabla \Psi) - 2\Psi \nabla^2 \Psi] = 0 \tag{14}$$

Equation (14) is the sum of the two parts, one linear in Ψ, and the other non-linear. The solution (12) converts the non-linear part into identical zero. On the other hand, this solution together with the dispersion relation (13) satisfies the linear part.. Therefore we have proven the following:

If the wave-like solution (12) satisfies Equation (8), then it is necessary and sufficient that it must be a solution of the following linear partial differential equation, the Schroedinger equation

$$[\, i\hbar \frac{\partial}{\partial t} + \frac{\hbar}{2m} \nabla^2] \Psi = 0 \tag{15}$$

Now we return back to the variable S according to $\Psi = \exp(iS/\hbar)$ and introduce the following dimensionless quantities: time $t = t/t_o$, spatial coordinates $\vec{R} = \vec{r}/L_o$, the parameter $\mathbf{h} = \hbar/S_o$, which we call the **Schroedinger number,** and the dimensionless action $\mathbf{S} = S/S_o$. Here $S_o = mL_o^2/t_o$, L_o is the characteristic length, and t_o is the characteristic time. As a result we transform (15) into the following dimensionless equation

$$\partial S/\partial t + \frac{1}{2}(\nabla S)^2 = (ih/2)\, \nabla^2 S \tag{16}$$

This equation is reduced to the Hamilton-Jacobi equation of the classical mechanics (or, equivalently the equation corresponding to the minimum fuzziness, as we discussed earlier) if its right hand side goes to 0. This possible only when the Schroedinger number **h** goes to zero. Therefore, at least for a free particle, this number serves as a measure of fuzziness of a microobject. Since $\hbar$ is a fixed number the limit $\mathbf{h} \to 0$ is possible only if $S_o \to \infty$,thus confirming our earlier assumption that action S represents a measure of fuzziness of a microobject. For a free particle, this means that with the decrease of S_O fuzziness of the particle increases.

Interestingly enough, the question of fuzziness (although not in these terms) was addressed in one of the first six papers on quantum mechanics written by E. Schroedinger[10]. He wrote "...the true laws of quantum mechanics do not consist of definite rules for the *single path*, but in

these laws the elements of the whole manifold of paths of a system are bound together by equations, so that apparently a certain reciprocal action exists between the different paths".

It has turned out that using the same reasoning as for a free particle we can easily derive the Schroedinger equation from the Hamilton-Jacobi equation for the case of a piecewise - constant potential as, for example, in the case of a square potential well. By replacing in the resulting Schroedinger equation the function Ψ by S according to (7), and introducing the dimensionless variables used in a study of a free particle, we obtain

$$\frac{\partial}{\partial t}\mathbf{S} + \frac{1}{2}(\nabla \mathbf{S})^2 + \mathbf{U} = (\mathrm{i}\mathbf{h}/2)\ \nabla^2\mathbf{S} \tag{17}$$

where $\mathbf{U} = U/S_o$ is the dimensionless potential. Once again the Schroedinger number serves as the sole indicator of the respective fuzziness, yielding the classical motion for $\mathbf{h} \to 0$.

A more complicated case of a variable potential $U(\vec{r},t)$ cannot be derived from the Hamilton-Jacobi equation with the help of the technique used so far, since there are no monochromatic complex wave solutions common for the non-linear Hamilton-Jacobi equation and the linear Schroedinger equation. Therefore, we postulate that the Schroedinger equation describing a case of an arbitrary potential $U(\vec{r},t)$, should have the same form as for a potential which is piece-wise constant. This postulate is justified by the fact that apart from the experimental confirmations in the limiting case of a very small Schroedinger number, $\mathbf{h} \to 0$ (minimum fuzziness) we recover the appropriate classical Hamilton-Jacobi equation. In what follows we will describe this process of recovering classical mechanics from quantum mechanics (which we dubbed defuzzification) in a different fashion that will require a study of a physical meaning of the function Ψ.

FUZZINESS AND THE WAVE FUNCTION Ψ

Earlier, by considering the Schroedinger number **h,** we saw that the action S represents some measure of fuzziness. Therefore, it is reasonable to expect that the function $\Psi = \exp(iS/\hbar)$ is also related to the measure of fuzziness. Since the fuzziness is measured by real-valued quantities (degree of membership, membership density), a possible candidate for such a measure

would be some function of various combinations of Ψ and Ψ^*. There is an infinite number of such combinations. However it is easy to demonstrate [11] that the Schroedinger equation is equivalent to the two nonlinear coupled equations with respect to the two real-valued functions constructed out of $\Psi\Psi^*$ and $(\hbar/2i)\mathrm{Ln}(\Psi/\Psi^*)$. Therefore, our choice of all possible real-valued combinations was reduced to only two functions. However, in the limiting transition to the classical case, only $(\hbar/2i)\mathrm{Ln}(\Psi/\Psi^*)$ is related to the classical velocity. As a result, we are left with only one choice, namely $\Psi\Psi^*$.

An appropriate way to see the physical meaning of $\Psi\Psi^*$ is to consider some not very involved specific example which can be easily reduced to a respective classical picture. We consider a solution of the Schroedinger equation for a free particle passing through a Gaussian slit[12]

$$\Psi = \sqrt{\frac{m}{2\pi i\hbar}}\frac{1}{\sqrt{T+t+\frac{i\hbar Tt}{mb^2}}}\exp\left\{\frac{im(v_0{}^2T+\frac{x^2}{t})}{2\hbar}+\frac{(\frac{m}{2\hbar t})^2(x-v_0t)}{\frac{im}{\hbar}(\frac{1}{T}+\frac{1}{t})-\frac{1}{b^2}}\right\} \tag{18}$$

where T is the initial moment of time, t is any subsequent moment of time, b is the half-width of the slit, $v_0 = x_0/T$, and x_0 is the coordinate of the center of the slit.

Using (18) we immediately find that $\Psi\Psi^*$ is

$$\Psi\Psi^* = (1/2\mathrm{phb}^2)[\,1 + t/T + \mathbf{h}^2\,]^{-1/2}\exp[\,-\,\mathbf{S}/\,(1 + t/T + \mathbf{h}^2\,] \tag{19}$$

where now $\mathbf{S} = (x - tv_0)/b^2$. Executing the transition to the case of a classical particle passing through an infinitesimally narrow slit we set both $\mathbf{h} \to 0$ and $b \to 0$. As a result, (19) will become the delta-function. Recalling that we define a classical mechanical particle as a fuzzy entity with a delta-like membership density, we arrive at the conclusion that the real-valued quantity $\Psi\Psi^*$ can be identified as the membership density for a microobject.

This allows one to ascribe to $\Psi\Psi^*dV$ the physical meaning of the degree of membership of a microobject in an infinitesimal volume dV (cf. to the analogous statement postulated in Ref.7).

This in turn implies a nice geometrical interpretation with the help of a generalization of Kosko's multidimensional cube. Any fuzzy set A (in our case a fuzzy state) is represented (see Fig.1 for a 2-D cube) by p.A inside this cube. Following Kosko, we use the sum of the projections of vector A onto the sides of the cube as the cardinality measure.

Let us consider the following integral

$$\int_{-\infty}^{\infty} \Psi\Psi^* dV = \operatorname{Lim}_{N\to\infty}\left(\sum_{i=1}^{N} \Psi_i\Psi_i^*\, dV_i\right) \tag{20}$$

If this integral is bounded, then we can normalize it. As a result, we can treat the right hand side of (20) as the sum of the projections of the "vector" $\int_{-\infty}^{\infty} \Psi\Psi^* dV$ onto the sides $\Psi_i\Psi_i^*\, \Delta V_i$ of the infinitely-dimensional parallelepiped. This allows us to represent the integral as the vertex A along the major diagonal of this parallelepiped.. According to the subsethood theorem [9], each side of the parallelepiped represents the degree of membership of the microobject (viewed as a deterministic fuzzy entity) in any given elemental volume dV_i built around a given spatial point x_i. According to the fuzzy theory the relative membership in two different spatial points x_i and x_j, that is $\Psi_i\Psi_i^*/\Psi_j\Psi_j^*$, is equal to the ratio of the respective numbers of the successful outcomes in a series of experiments aimed at locating the microobject (or rather its part) at the respective elemental volumes. Hence we can conclude that the membership density at a certain point is proportional to the number of successful outcomes in repeated experiments aimed at locating the fuzzy microobject at the respective elemental volumes.

If the integral on the right-hand side of (20) is divergent, this does not change our arguments, since $(\Psi\Psi^*)$ is a measure of the successful outcomes in a series of experiments that do not depend on the convergence of the integral. Thus we see that the fuzziness, via its membership density, dictates the number of successful outcomes in experiments aimed at locating the fuzzy microobject. Continuing this line of thought we see that any physical quantity associated with the fuzzy microobject is not tied to a certain spatial point. This indicates a need to introduce a process

of defuzzification with the help of the membership density which would serve as the "weight" in this process. Such defuzzification is different from what is usually understood by this term, that is a process of "driving" a fuzzy point to a nearest vertex of a hypercube. Instead, we take the degree of membership $(\Psi\Psi^*)_i\Delta V_i$ at each vertex of the infinite-dimensional parallelepiped and multiply it by the value of the physical quantity at the respective point x_i. Summing over all these products results in the averaged (defuzzified) value of the quantity.

Thus, instead of averaging over the distribution of random quantities, we introduce the defuzzification of deterministic quantities. Mathematically both processes are identical, but physically they are absolutely different. We do not need the probabilistic interpretation of the wave function Ψ, which implies that there is another, more detailed level of description that would allow us to get rid of uncertainties introduced by randomness. Now it is clear that within the framework of the fuzzy interpretation, we cannot get rid of the uncertainties *intrinsic* to fuzziness and not connected to randomness. From this point of view quantum mechanics does not need any hidden variable to improve its predictions. They are precise within the framework of the fuzzy theory.

Moreover, since quantum mechanics is a linear theory, one can speculate that according to fuzzy approximation theorem(13), the linearity and fuzziness of quantum mechanics are the best tools to approximate (with any degree of accuracy) any macrosystem (linear or nonlinear). The linearity of quantum mechanics is responsible for the uncertainty relations which are present in any linear system. Therefore, (as was demonstrated long ago(6)) these relations enter quantum mechanics even before any concept of measurement.

Let us consider the membership density of a free microobject (a progenitor of a classical free particle). It is obvious that $\Psi\Psi^*$=const. This means that the relative degree of membership for any two points in space is 1. In other words the free microobject is "everywhere", which is the same property as say, a 3-D standing wave has. This example shows that the wave-particle duality is not a duality at all, but simply an expression of the fuzzy nature of things quantum. In fact, we can even go so far as to claim that the complementarity principle is a product of a compromise between the requirements of the bivalent logic and the results of quantum experiments.

Within the framework of the fuzzy approach there is no need to require complementarity, since the logic of a fuzzy microobject transcends the description of its properties in terms of either-or, and as a result is much more complete, probably the most complete description under the given experimental results.

It has turned out that the membership density has something more to offer than simply a degree to which a fuzzy microobject has a membership in a certain elemental volume dV. In fact, using expansion of the wave amplitude (we could call it **fuzziness amplitude**) Ψ in its orthonormal eigenfunctions Ψ_k and assuming that the integral in (20) is bounded, we write the well-known expression

$$\int_{-\infty}^{\infty} \Psi\Psi^* dV = \sum_{k=1}^{\infty} a_k a_k^* = 1 \tag{21}$$

Equation (21) allows a very simple geometric interpretation with the help of a (N-1) - dimensional simplex. A fuzzy state Ψ is represented as a point A on the boundary of this simplex (Fig. 2 shows this for a 1-D simplex, k=1,2). Its projections onto the respective axes corresponds to the values $a_k a_k^*$. Now applying the subsethood theorem, we interpret the values of $a_k a_k^*$ as the degree to which the state A is contained in a particular eigenstate k. Using Fig. 2 we can clearly see that $A \cap B = B$, $A \cap C = C$. Moreover, the same figure shows us that the lengths of projections of A onto the respective axes (namely,OA and OC) are nothing more than the cardinality sizes $M(A \cap B) = a_1 a_1^*$ and $M(A \cap C) = a_2 a_2^*$. On the other hand, the cardinality size of A is $M(A) = 1$. Therefore the respective subsethood measures are $S(A,B) = a_1 a_1^*/1$ and $S(A,C) = a_2 a_2^*/1$. At the same time, both of these measures provide a number of successful outcomes, that is, detections of the respective states k=1 or k=2 in the repeated experiments.

The picture which we discussed corresponds to a particular case when a state A has a wave (fuzziness) amplitude Ψ which corresponds to the pure state. At the same time, it is general

enough to describe a mixed state characterized by what is called in the probability interpretation the density matrix $r(x',x)$ The integral of $r(x',x)$ over all x's yields the following sum:

$$\sum_{k=1}^{\infty} a_{kk}$$

which is the generalization of a measure of containment of the fuzzy state A in the discrete states k. By preparing a certain state, which is now understood to be a fuzzy entity, we fix the frequencies of the experimental realizations of this fuzzy state in its substates k. If the fuzzy state A undergoes a continuous change, which corresponds in Fig. 2 to motion of p. A along the straight line, then its subsethood in any state k also changes. This implies the following: if the eigenfunctions of a fuzzy set stay the same, the degree to which the respective eigenstates represent the fuzzy state varies. This variation can occur continuously despite of the fact that the eigenstates are discrete.

This indicates an interesting possibility that quantum mechanics is not necessarily tied to the Hilbert space. Such a possibility was mentioned long ago by J. von Neumann [14], and quite recently was addressed by C. Wulfman.[15]. One of the hypothetical applications of this idea is to use quantum systems as an infinite continuum state machine in a fashion which is typical for a fuzzy system : small continuous changes in the input from some ugly nonlinear system will result in small changes at the output of the quantum system which in turn can be correlated with the input to produce the desired result.

Concluding our introduction to a connection of fuzziness and quantum mechanics we prove a statement which can be viewed as the generalized Ehrenfest theorem. We will demonstrate that defuzzification of the Schroedinger equation, with the help of the membership density $\Psi\Psi^*$, will yield the Hamilton-Jacobi equation. This will provide *a posteriori* derivation of the Schroedinger equation for an arbitrary potential $U(\vec{r},t)$. We assume that the fuzzy amplitude $\Psi \rightarrow 0$ as $r \rightarrow \infty$, and rewrite the Schroedinger equation as follows

$$(\hbar/i)\frac{\partial}{\partial t}\mathrm{Ln}\Psi + (\hbar^2/2m)(\nabla \mathrm{Ln}\Psi)^2 + V \qquad (22)$$

Integrating (22) with the weight $\Psi\Psi^*$ (that is "defuzzifying" it), we obtain

$$\int \Psi\Psi^*[\,(\hbar/i)\frac{\partial}{\partial t}\mathrm{Ln}\Psi + (\hbar^2/2m)\,(\nabla\,\mathrm{Ln}\Psi)^2 + V]\,d^3x = 0 \tag{23}$$

Integrating the second term by parts and taking into account that the resulting surface integral vanishes because $\Psi \to 0$ at infinity, we obtain the following equation

$$< \partial S/\partial t> + (1/2m)<(\nabla S)(\nabla S^*)> + <V> = 0$$

where < > denote defuzzification with the weight $\Psi\Psi^*$, and $S = (\hbar/i)\mathrm{Ln}\Psi$. This equation is analogous to the classical Hamilton-Jacobi equation (4).

The generalized Ehrenfest theorem shows that the classical description is true only on a coarse scale generated by the process of "defuzzification", or measurement. The "classical measurement" corresponds to the introduction of a non-quantum concept of the potential $U(\vec{r},t)$ serving as a short-hand for the description of a process of interaction a microobject (truly quantum object) with a multitude of other microobjects. This process destroys a pure fuzzy state (a constant fuzziness density) of a free quantum "particle".

Paraphrasing A. Peres [(16)], we can say that a classical description is the result of our "sloppiness", which destroys the fuzzy character of the underlying quantum mechanical phenomena. This means that, in contradistinction to Peres, we consider them "fuzzy" in a sense that the respective membership distribution in quantum mechanics does not have a very sharp peak characteristic for a classical mechanical phenomena. Note that we exclude from our consideration the problem of the classical chaos, assuming that our repeated experiments are carried out under the absolutely identical conditions.

In a final part of our paper we consider a possible use of quantum fuzziness in optical computing.

OPTICAL COMPUTING

Essentially all universal computing machines are based on the Turing paradigm, that is on the following:

- input data
- input instructions (according to some program based on some already known algorithm)
- availability of one or more physical devices necessary to perform each step

Note that the Turing paradigm presupposes the knowledge of an algorithm, and as such cannot be used for serendipity problems.

This paradigm is so simple and so deeply embedded in our thinking that alternatives appear almost inconceivable. However one of the authors (J.C.), has described computers (Refs., 17-25) which avoid the third step in the Turing paradigm. That step could be very costly from the point of view of resources consumption. If a physical device (in this case a gate) is dissipative (irreversible) then the third step is costly in energy terms because of the dissipation of energy. On the other hand if there was a possibility to make a lossless gate, the accumulation and processing of unnecessary information ("garbage") during intermediate calculations could eventually lead to an unreasonable high expenditure of time. Here we can consider time as one of the resources that we prefer to spent efficiently. This compels us to look at the "garbage" closer.

Garbage is information generated by computers en route to solving problems, which by itself, is of no value as soon as we get the answer. If for example we have to calculate $\sum_{i=1}^{100} i$, then one of the ways to solve the problem is to do the following

$$1 + 2 = 3$$

$$3 + 3 = 6$$

$$6 + 4 = 10$$

$$\vdots$$

$$4950 + 100 = 5050$$

where numbers 3,6, 10, ..., 4950 can be considered as garbage.

A non-dissipative garbage collection can be identified as one of the major problems in utilizing the potential advantages of non-dissipative logic gates, even if it will be possible to build them. And yet the time spent in collecting the garbage could present a formidable problem. On the other hand, dissipative computer logic gates, if used in the process of " garbage collection", destroy information and, as a result, increase entropy. The respective entropy increase accompanying a destruction of one bit of information is

$$\Delta S \geq k\mathrm{Ln}\ 2$$

and the respective quantity of energy expended in this process is

$$\Delta E = T\Delta S \geq kT\mathrm{Ln}\ 2$$

where k is the Boltzmann constant. Information-conserving, reversible (non-dissipative) logic gates are possible in principle[26]. This means that $\Delta S = 0$, and respectively $\Delta E = 0$. However, in practice all such gates to date exhibit $\Delta E > kT\mathrm{Ln}2$.

However if we look at quantum mechanics, the entropy increase occurs only when defuzzification (detection) occurs. If it would be possible to avoid defuzzification of the garbage bits then it would mean a lossless (unitary) propagation of a wave function. This provides an obvious advantage of such a computer as compared with a computer built on the basis of dissipative gates. Moreover, it was shown previously (Refs. 17-25), that it is often possible even to avoid "premature" measurements in analog or digital quantum computers. The defuzzification which could be made, but did not, would have resulted in garbage. This means that some garbage can be totally avoided, thus providing another advantage of a quantum computer even compared to a Turing machine built with lossless gates. Which leads us to the following question: how much garbage can be avoided?

The answer to this question is to some extent surprising. Before proceeding with the answer we want to address the related problem. How much garbage can we create with physical systems? The answer is a finite amount. The upper limit of this amount can be easily evaluated by using very simple arguments. Suppose that we have a certain mass m which we want to use to a maximum possible degree in a computer. The maximum available energy in this case is given by

the rest energy mc^2. Further suppose that we operate in an ideal environment at zero temperature, $T = 0$, during a time interval Δt. The maximum energy resolution is given by the uncertainty principle

$$\Delta E \, \Delta t \geq \hbar /2$$

This means that the maximum number of distinguishable states which can be obtained in the processes of defuzzification is

$$N_{max} = E/\Delta t = 2mc^2 \, \Delta t/ \hbar$$

If we would have installed N intermediate gates to detect all these states then the number of garbage bits could not exceed N_{max}. Let us consider a concrete example by taking some absurd values:

$$m = 1 \text{ Earth mass}$$

$$\Delta t = 10^6 \text{ years}$$

The resulting number of garbage bits is an astronomical number $N_{max} \sim 10^{80}$ and not less astronomical amount of time spent in detecting the garbage bits. However, amazingly enough, we can easily avoid more garbage than generated by this hypothetical computer. Let us consider 50 plates separated from each other by a distance 2f, where f is the focal distance of lenses inserted between these plates. Each plate has 50 holes. We look for the brightness at some point in the 51-st plate. This would require (if we assume that a photon has a membership in all the path connecting all the holes) about $50^{50} \sim 10^{82}$ calculations. It seems that it is impossible to find the brightness via the Turing paradigm within the reasonable time even if we use the lossless gates. However, the answer is found experimentally by performing crude quantum calculations, and it is produced in an amazingly short time: a few nanoseconds.

How can this be? The answer is very straightforward; we never defuzzify the garbage information on all 50^{50} paths, that is the defuzzification process before its final phase, detection, was never detected (recorded), nor interfered in any way thus avoiding additional generation of N bits of "expendable" information. In this sense this computing device is garbage-free.

In some quantum computers there is no limit to the avoidable garbage, and avoided garbage need not to be cleaned. This means that there is no upper limit to the effective speed of quantum computers measured relative to conventional Turing-equivalent computers in which the garbage is generated by default via use some sort of gates (dissipative or non-dissipative).

What happens can be explained in fuzzy terms. Before measurement (defuzzification) of the final result, the optical activity had an actual, fuzzy presence in all the paths. The defuzzification of the final result looses all that garbage information but retains only the required results. In terms of a wavefunction we can say that Ψ is prepared in such a way that $|\Psi|^2$ provides the desired result, and the undetectable quantum phase corresponds to the connection to globality of all the paths. The phase and its global information are lost when we get the "final" product of defuzzification, that is, $|\Psi|^2$. Since we are interested only in this product, the lost information (which contributed to the generation of the product) can be viewed as "garbage". It seems that such an explanation is more congenial than the one in which the garbage avoided in this universe is accumulated in all parallel universes.

CONCLUSION

This work represents a continuation of our previous published (Ref. 2) effort to understand quantum mechanics in terms of the fuzzy logic paradigm. We regard reality as intrinsically fuzzy. In spatial terms, this is often called nonlocality. Reality is nonlocal temporarily as well, which means that any microobject has membership(albeit to a different degree) in both the future and the past. In this sense one might define present as the time average over the membership density.

Measurement is defined as a continuous process of defuzzification whose final stage detection is inevitably accompanied by a dramatic loss of information through emergence of locality, or crispness in fuzzy logic terms.

We attempted to provide a description of quantum mechanics in terms of deterministic fuzziness. It is understood that this attempt is inevitably incomplete and has many features which can be improved, extended, or corrected. However we hope that this work will inspire other people

to start looking at the quantum phenomena through "fuzzy" eyes and maybe something practical (apart from removing wave-particle duality and complementary mysteries) will come out of this.

ACKNOWLEDGMENTS. The authors thank J.Hyatt for reading the manuscript. One of the authors (A.G.) wishes to thank V. Panico for very illuminating discussions which helped to shape this work and for reading the manuscript. H.J.C.'s work was supported by the Air Force Office of Scientific Research. Both authors thank the reviewer for very constructive remarks.

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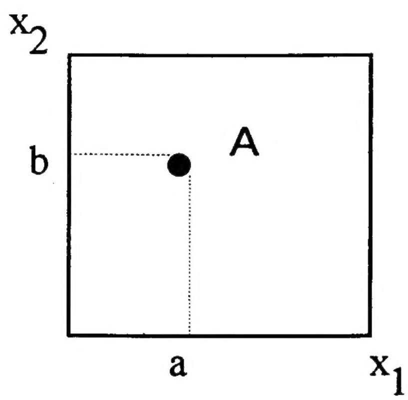

FIG.1

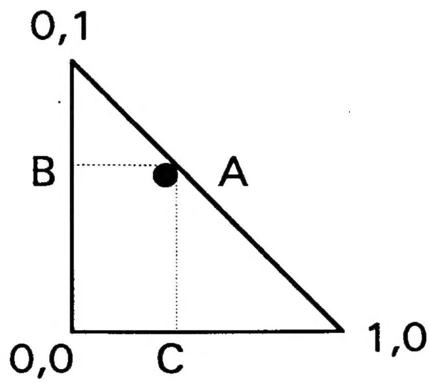

FIG.2

FIGURE CAPTIONS

Fig.1 Geometrical interpretation of fuzzy sets.The fuzzy subset A is a point in the unit 2-cube with coordinates a and b.The cube consists of all possible fuzzy subsets of two elements x_1 and x_2.

Fig.2 Representation of a quantum mechanical state A as a point in a 1-dimensional fuzzy simplex.